5.5 Applications of Exponential Functions

▼ Periodic Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

P-principle or initial investment
r-annual interest rate
n-number of compounding in one year
t-time in years
A-Accrued amount

▼ Continuous Compound Interest

$$A = Pe^{rt}$$

P-principle or initial investment
r-annual interest rate
t-time in years
A-Accrued amount

▼ Example: Find the Time to Double an Investment

Suppose that a bank offers you an account that pays 7% annually compounded continuously. If you plan to deposit $1000, how long will it take for you money to double? Round your answer to the nearest year.

▼ Example: Find the Interest Rate to Triple an Investment

Suppose that broker tells you that it will take 15 years to triple your money on an investment that is compounded continuously. What is the annual interest rate that you will be earning? Round your answer to three decimal places.

▼ Population Growth or Decay

$$P(t) = P_0 e^{kt}$$

P_0-initial amount

k-growth or decay rate

t-time

P-Population after time

▼ Example: Insect Population

The size P of a certain insect population at time t (in days) obeys the function $P(t) = 300e^{0.05t}$

- Determine the number of insects at $t = 0$ days.
- What is the growth rate of the insect population?
- What is the population after 10 days?

- When will the insect population double?

▼ Doubling Time, Tripling Time, Half-Life

- Doubling-time is the time it takes for a quantity to double in size.
- Tripling-time is the time it take for a quantity to triple in size.
- Half-life is the time it takes for a quantity to decay to half of it's size.

These establish a relationship between a starting and ending amount over a period of time. With this information, you can adjust the formula for the given time and a known accrued amount (double, triple or half) then solve for the rate.

▼ Example: City Population Doubles in Size

The population of a city follows the exponential growth model and it is known that the population doubles in size over 13 months. The current population is 1,234, what will the population be in 5 years from now?

▼ Law of Uninhibited Growth or Decay

$$A(t) = A_0 e^{kt}$$

A_0-initial amount

k-growth or decay rate

t-time

A-Accrued amount

▼ Example: Half-life of a Chemical Element

The chemical element has a half-life of approximately 4 days. If 236 grams are present now, how much will be present in 60 days?

▼ Example: Half-life of Caffeine

The amount of caffeine in the human body has a half life of 4.9 hours to 6 hours depending on the person. Assuming the caffeine half life for a particular person is 5.5 hours find the decay rate of caffeine and build an exponential model for the decay rate. Find the amount of caffeine left in the body 2 hours after drinking a can of monster energy drink which contains 86 milligrams of caffeine. Round your answer to three decimal places.