5.4A Exponential Equations

- Solving Exponential Equations
  - Method: Relating the Bases or One-to-One Property
    - Method of Relating the Bases in Symbols and Words
      1. Write the equation in a form where the bases are the same on both sides. $[b^u = b^v]$
      2. Use the one-to-one property which states if the bases are the same then the exponents are equal. $[u = v]$
    - Examples of Solving Exponential Equations by Relating the Bases
      - Example 1: $4^{2x+1} = 8^x$
      - Example 2: $e^{-x^2} = (e^x)^2 \cdot \frac{1}{e^3}$
      - Example 3: $3^{7x} = 9^{2x-5}$

▼ Example 4: $3^{5x} \cdot 9^{x^2} = 27$

▼ Example 5: $5^x - 2 = 23$

▼ Example 6: $3^{x+1} = 9^{5x}$

▼ Example 7: $7^{x^2+3x} = \frac{1}{49}$

▼ Example 8: $9^{x^2} = 3^{-5x-2}$

- Example 9: $5^{2x^2+3x} = 25^{6-x}$

- Example 10: $\left(e^{-x}\right)^2 = \frac{e^x}{e^2}$

- Method: Convert to Logarithmic Equation
  - Method of Converting to a Logarithm in Symbols and Words
    1. Recognize an exponential equation. $y = b^x$
    2. Rewrite the exponential equation as a logarithmic equation using the following equivalence. $[\log_b y = x]$
    3. Solve the remaining equation to find solutions.
  - Examples of Solving Exponential Equations
    - Example 1: $2^x = 5$

    - Example 2: $8 \cdot 3^x = 5$

    - Example 3: $5^{x-2} = 3^{3x+2}$

▼ Example 4: $3^x = 8$

▼ Example 5: $10^x = 1500$

▼ Example 6: $5^{x-3} = 137$

▼ Example: 7: $7^{2x+1} = 3^{x+5}$

▼ Example 8: $500e^{3x} + 40 = 1040$

▼ Method: Take the Log of Both Sides

▼ Method of Take the Log of Both Sides in Symbols and Words

1. Recognize an exponential equation. $[y = b^x]$
2. Use the one-to-one property for logarithms and take the natural log of both sides. $[\ln y = \ln b^x]$
3. Use the power rule for logarithms which allows you to move the power inside the logarithm to become the coefficient of the logarithm. $[\ln y = x \ln b]$
4. Solve for the variable by dividing. $\left[\frac{\ln y}{\ln b} = x\right]$

$$y = b^x$$
$$\ln y = \ln b^x$$
$$\ln y = x \ln b$$
$$\frac{\ln y}{\ln b} = \frac{x \ln b}{\ln b}$$
$$\frac{\ln y}{\ln b} = x$$

▼ Examples of Solving Exponential Equations

▼ Example 1: $2^x = 5$

▼ Example 2: $8 \cdot 3^x = 5$

▼ Example 3: $5^{x-2} = 3^{3x+2}$

▼ Example 4: $3^x = 8$

▼ Example 5: $10^x = 1500$

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▼ Example: 7: $7^{2x+1} = 3^{x+5}$

▼ Example 8: $500e^{3x} + 40 = 1040$

▼ Method: Quadratic in Form or u-substitution

▼ Method of Quadratic in Form in Symbols and Words

1. Recognize an equation is quadratic in form $[a \cdot w^{2x} + b \cdot w^x + c = 0]$

2. The substitution $u = w^x$ turns the equation into a quadratic equation.
   $[au^2 + bu + c = 0]$
3. Solve the quadratic equation by factoring, the square root method, completing the square or the quadratic formula. $[u = l$ or $u = m]$
4. Return to the original equation to the original equation using the same substitution as before. $[w^x = l$ or $w^x = m]$
5. Solve the remaining exponential equation.

▼ Examples of Solving Exponential Equations by u-substitution

  ▼ Example 1: $e^{2x} - 2e^x - 3 = 0$

  ▼ Example 2: $4^x - 2^x - 12 = 0$