

5.3 Properties of Logarithms

▼ Properties with 1

- ▼ Example 1: Evaluate a logarithmic expression without a calculator

$$\log_2 1$$

- ▼ Example 2: Evaluate a logarithmic expression without a calculator

$$\log_7 7$$

- ▼ $\log_b b = 1$

A logarithm in which the base is the same as what is substituted inside the logarithm is one.

- ▼ $\log_b 1 = 0$

A logarithm in which one is substituted inside the logarithm is zero.

▼ Condensing and expanding properties

- ▼ Product Rule for Logarithms

$$\log_b(M \cdot N) = \log_b M + \log_b N$$

- Expanding: the expression on the left has a product inside the logarithm. You can rewrite as the sum of logarithms of each factor.
- Condensing: the expression on the right is the sum of logarithms with the same base. You can rewrite as a single logarithm of the product of the expressions inside each logarithm.

- ▼ Quotient Rule for Logarithms

$$\log_b \left(\frac{M}{N} \right) = \log_b M - \log_b N$$

- Expanding: the expression on the left has a quotient inside the logarithm. You can rewrite as the difference of logarithms of the numerator and denominator.
- Condensing: the expression on the right is the difference of logarithms with the same base. You can rewrite as a single logarithm of the quotient of the expressions inside each logarithm.

▼ Power Rule for Logarithms

$$\log_b(M^p) = p \cdot \log_b M$$

- Expanding: the expression on the left has a power inside the logarithm. You can rewrite by multiplying the power by the logarithm of the base.
- Condensing: the expression on the right is a product of p and a logarithm. You can rewrite as a logarithm with the p as the exponent of the expression inside the logarithm.

▼ Examples of Expanding

▼ Example 1: $\log_2(8x)$

▼ Example 2: $\log\left(\frac{100}{y}\right)$

▼ Example 3: $\ln(x^3)$

▼ Example 4: $\log_3 \left(\frac{27x^3}{y^5 \sqrt{z}} \right)$

▼ Examples of Condensing

▼ Example 1: $\log_3 7 + \log_3 x$

▼ Example 2: $\log_5 50 - \log_5 2$

▼ Example 3: $2 \log_3 x$

▼ Example 4: $3 \ln x + 4 \ln(x + 1) - \frac{1}{2} \ln(x - 1) - 7 \ln(x^2 - 9)$

▼ Inverse properties

▼ $\log_b b^p = p$

A logarithm in which the expression inside is exponential and has the same as the base of the logarithm is the expression in the exponent. This

happens because of the cancellation property of inverses. $(f^{-1} \circ f)(x) = f^{-1}(f(x)) = x$

▼ $b^{\log_b M} = M$

An exponential expression in which the exponent has a logarithm of the same base is the expression inside the logarithm. This happens because of the cancellation property of inverses. $(f \circ f^{-1})(x) = f(f^{-1}(x)) = x$

▼ Examples applying inverse properties

▼ Example 1: $\log_3 3^{2x-5}$

▼ Example 2: $6^{\log_6(3-x)}$

▼ Example 3: $\ln e^{0.05t}$

▼ Example 4: $\ln(2e^{3k})$

▼ Change of base

$$\log_b M = \frac{\log_a M}{\log_a b} = \frac{\log M}{\log b} = \frac{\ln M}{\ln b}$$

Most calculators are only programmed to simplify common logarithms (base 10) and natural logarithms (base e). You can use this formula to change the base to any base of your choosing. In the formula above, a is the base that you choose to change to. Most of the time we will be changing to base 10 or e so that we can use the calculator to evaluate.

▼ One-to-one Property for Logarithms or the Property of Equality

$$\begin{aligned}\log_b u &= \log_b v \\ u &= v\end{aligned}$$

- Requires the logarithmic equation to have a logarithm with the same base on both sides.
 - When the bases of the logarithms are the same the expressions inside must be equal because of the one-to-one property of logarithmic functions.
 - Solve the remaining equation by isolating x .
 - Check proposed solutions to be sure they are in the domain of all logarithmic expressions.
- ▼ Examples solving logarithmic equations with the property of equality
- ▼ Example 1: $\log_5(x - 7) = \log_5(2)$

▼ Example 2: $2 \log_7 x = \log_7 64$

▼ Example 3: $\log(x - 9) = \log(x + 4) + \log 3$

▼ Example 4: $\ln x + \ln(x - 4) = \ln(x + 6)$