5.2 Logarithmic Functions

- Definition: Logarithmic Function

  The logarithmic function to the base b, where b>0 and not equal to one, is denoted by

  $$y = \log_b x$$

  (read as "y is the logarithm base b of x") and is defined by

  $$y = \log_b x$$

  if and only if

  $$x = b^y$$

- Examples: Convert an exponential equation to a logarithmic equation
  - Example 1: Convert an exponential equation to a logarithmic equation

    $1.2^3 = m$
  - Example 2: Convert an exponential equation to a logarithmic equation

    $a^4 = 24$
  - Example 3: Convert an exponential equation to a logarithmic equation

    $10^4 = 10000$
  - Example 4: Convert an exponential equation to a logarithmic equation

    $e^p = 9$
- Examples: Convert a logarithmic equation to an exponential equation
  - Example 1: Convert the logarithmic equation to an exponential equation

    $\log_b 4 = 5$
  - Example 2: Convert the logarithmic equation to an exponential equation

    $\log_3 5 = c$

- Example 3: Convert the logarithmic equation to an exponential equation

  $\log 1000 = 3$

- Example 4: Convert the logarithmic equation to an exponential equation

  $\log 1000 = 3$

- Common Logarithms

  Special notation is used for logarithms with base 10. Our number system has digits with place values of powers of 10. When no base is specified in the superscript base 10 is the assumed base.

$$\log x = \log_{10} x$$

- Natural Logarithms

  Special notation is used for logarithms with base e. Base e is observed in many formulas involving exponential functions. The symbol "$\ln$" is reserved for a logarithm with base e.

$$\ln x = \log_e x$$

- Examples: Evaluate a logarithmic expression without a calculator

  - Example 1: Evaluate a logarithmic expression without a calculator

    $\log_2 16$

  - Example 2: Evaluate a common logarithmic expression without a calculator

    $\log 100$

  - Example 3: Evaluate a logarithmic expression without a calculator

    $\log_3 \left(\frac{1}{27}\right)$

▼ Example 4: Evaluate a natural logarithmic expression without a calculator

$\ln\left(\frac{1}{e}\right)$

▼ Example 5: Evaluate a logarithmic expression without a calculator

$\log_{25} 5$

▼ Example 6: Evaluate a logarithmic expression without a calculator

$\log_2 1$

▼ Example 7: Evaluate a logarithmic expression without a calculator

$\log_7 7$

▼ Example 8: Undefined logarithmic expression

$\log_b 0$

▼ Example 9: Undefined logarithmic expression

$\log_b(-1)$

▼ Properties with 1

- $\log_b b = 1$

  A logarithm in which the base is the same as what is substituted inside the logarithm is one.

- $\log_b 1 = 0$

  A logarithm in which one is substituted inside the logarithm is zero.

- Domain of logarithms

  If $y = \log_b g(x)$ is a logarithmic function. The logarithm if a negative is undefined. the logarithm of zero is undefined. The logarithm is defined for positive values. Positive values are greater than zero. To find where the function is defined, create an inequality using the expression inside the logarithm set greater than zero and solve.

  $$g(x) > 0$$

- Examples: Find the domain of a logarithmic function

  - Example 1: Find the domain of the logarithmic function

    $$f(x) = \log_b(x+3)$$

  - Example 2: Find the domain of the logarithmic function

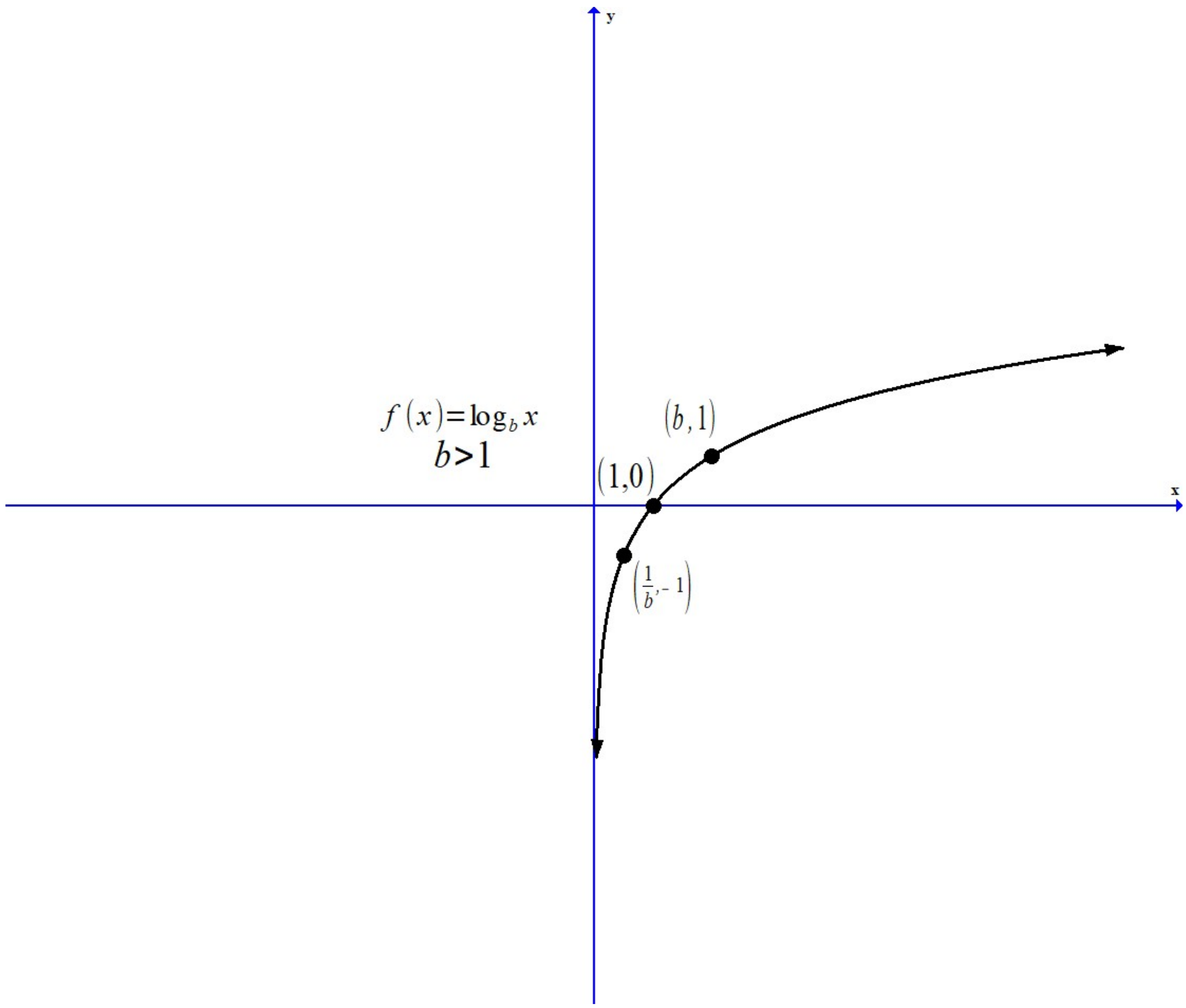
    $$f(x) = \log_b(2-x)$$

  - Example 3: Find the domain of the logarithmic function

    $$f(x) = \log_b |x|$$

▼ Properties of a Logarithmic Function (b>1)

$$f(x) = \log_b x; b > 1$$

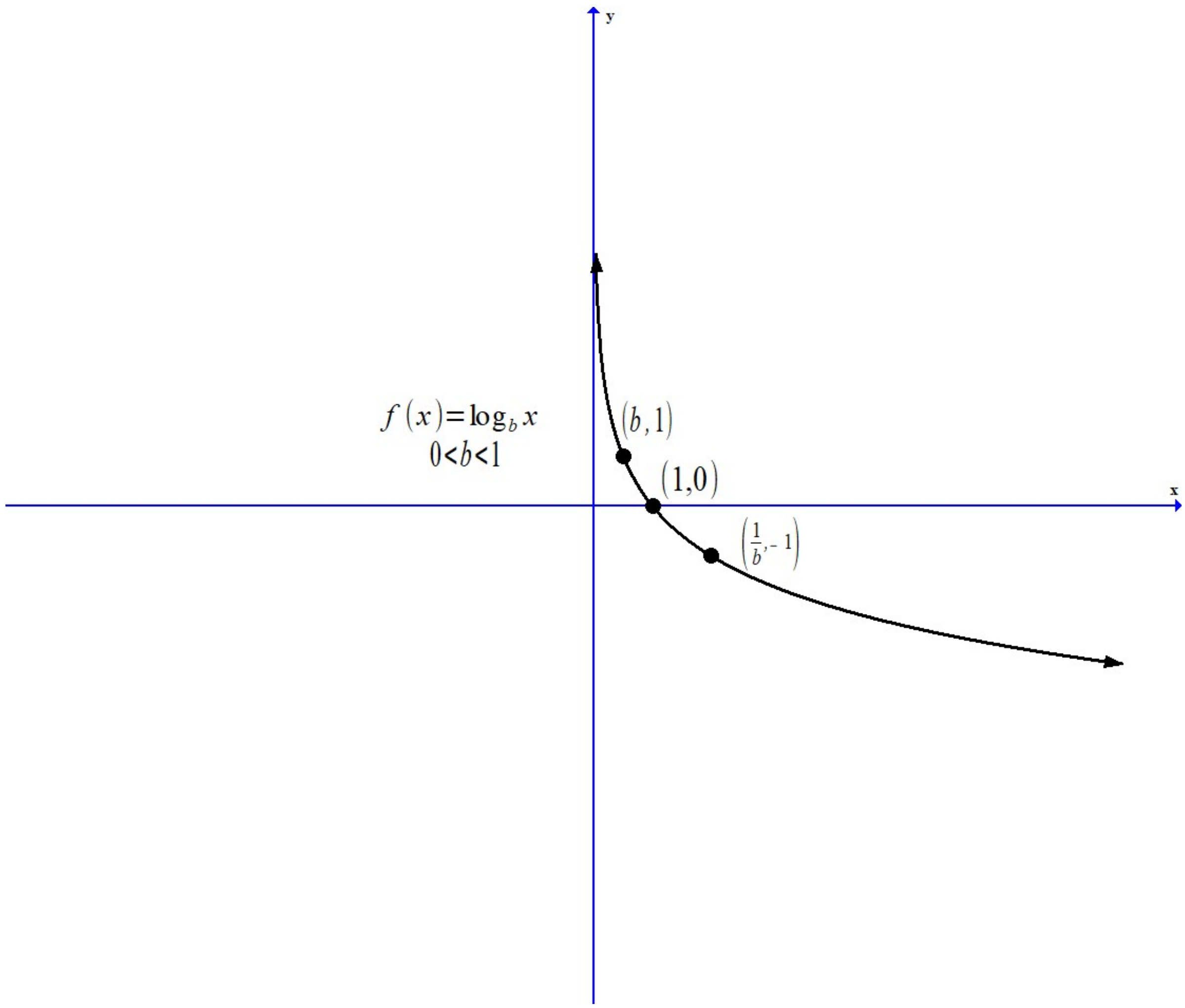


1. The domain is the set of all positive numbers and the range is the set of all real numbers.
2. There are no y-intercepts and the x-intercept is 1.
3. The y-axis $(x = 0)$ is a vertical asymptote as x approaches zero from above zero.

   $f(x) \to -\infty$ as $x \to 0^+$
4. $f(x)$ is an **increasing** function and is one-to-one.
5. The graph of *f* is smooth and continuous, with no corners or gaps.

▼ Properties of a Logarithmic Function (0<b<1)

$$f(x) = \log_b x; 0 < b < 1$$



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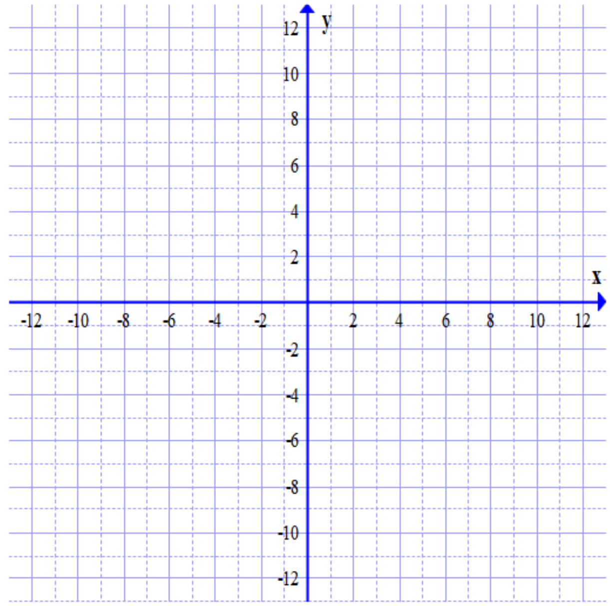
3. The y-axis $(x = 0)$ is a vertical asymptote as x approaches zero from above zero.

   $f(x) \to \infty$ as $x \to 0^+$
4. $f(x)$ is a **decreasing** function and is one-to-one.
5. The graph of *f* is smooth and continuous, with no corners or gaps.

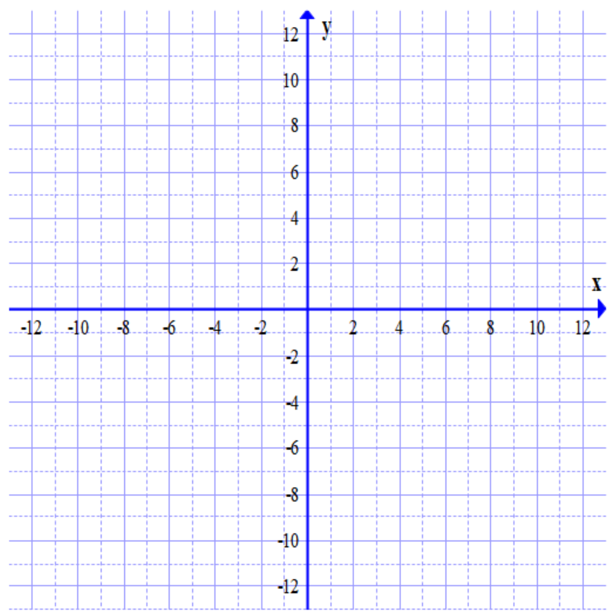
▼ Example: Graph a logarithmic function base 2 by plotting points

x	y	(x,y)



▼ Example: Graph a logarithmic function base 1/2 by plotting points

x	y	(x,y)



▼ Example: Graph $f(x) = -\log_2(x-3) + 2$ by Transformations

Base Function: ________________

List Transformations:

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