5.1A Exponential Functions

▼ Definition of an Exponential Function

An **exponential function** is a function of the form

$$f(x) = b^x$$

where x is any real number and $b > 0$ such that $b \neq 1$.

▼ Definition of the Natural Base

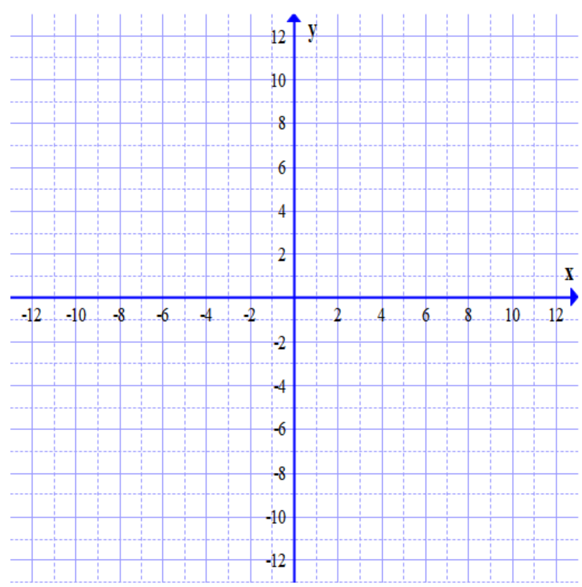
The number e is an irrational number and is defined by the expression $\left(1+\frac{1}{n}\right)^n$ as $n \to \infty$.

The expression $\left(1+\frac{1}{n}\right)^n \to e \approx 2.718$ as $n \to \infty$.

▼ Examples of Graphing Basic Exponential Functions

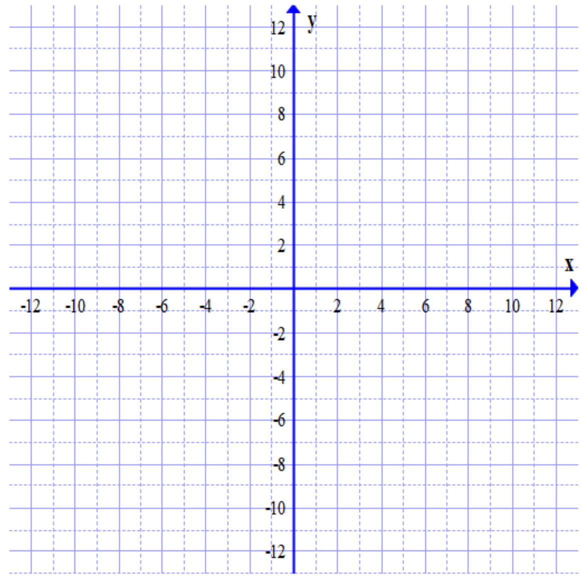
▼ Example 1: Graph $y = 2^x$ by plotting points

x	y	(x,y)



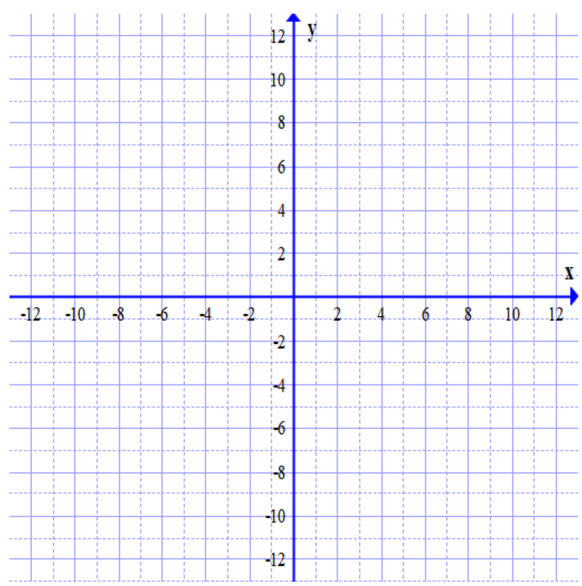
▼ Example 2: Graph $y = \left(\frac{1}{3}\right)^x$ by plotting points

x	y	(x,y)

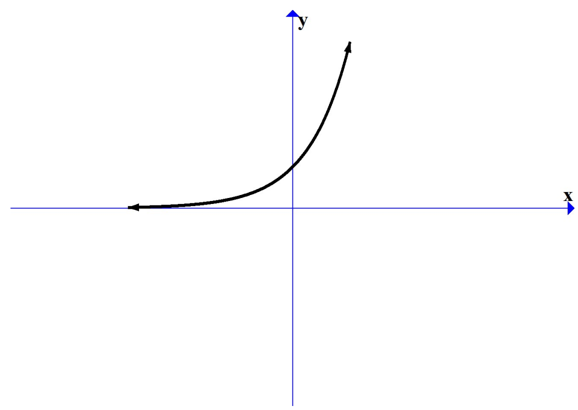


▼ Example 3: Graph $y = e^x$ by plotting points

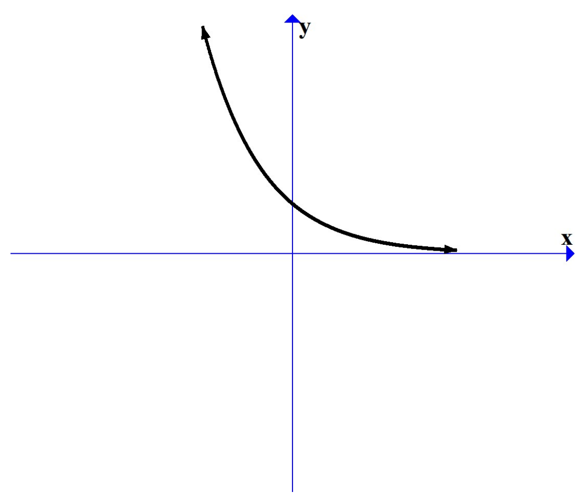
x	y	(x,y)



- Characteristics of an Exponential Function in the form $f(x) = b^x$
  - Domain: $(-\infty, \infty)$
  - Range: $(0, \infty)$
  - Intersects the y-axis at $(0, 1)$
  - One-to-one function
  - Horizontal asymptote at $y = 0$
  - If $b > 1$
    - Increasing on the domain
    - $b^x \to \infty$ as $x \to \infty$ (y approaches infinity as x approaches infinity)
    - $b^x \to 0$ as $x \to -\infty$ (y approaches 0 as x approaches negative infinity)



  - If $0 < b < 1$
    - Decreasing on the domain
    - $b^x \to 0$ as $x \to \infty$ (y approaches 0 as x approaches infinity)
    - $b^x \to \infty$ as $x \to -\infty$ (y approaches infinity as x approaches negative infinity)



▼ Examples: Graphing Transformations

▼ Example 1: Graph $f(x) = -2^{x-3} - 5$ by Transformations

Base Function: ________________
List Transformations:

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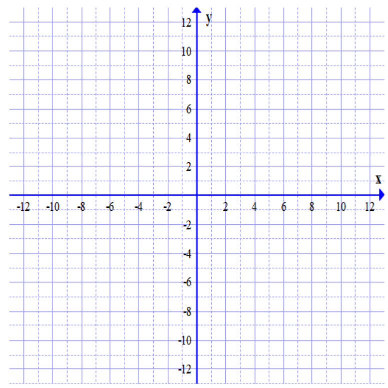

▼ Example 2: Graph $f(x) = -\left(\frac{1}{3}\right)^{x+5} + 2$ by Transformations

Base Function: ________________
List Transformations:

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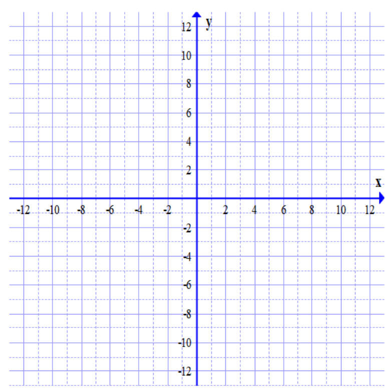

▼ Example 3: Graph $f(x) = -e^{-x}$ by Transformation

Base Function: ________________
List Transformations:

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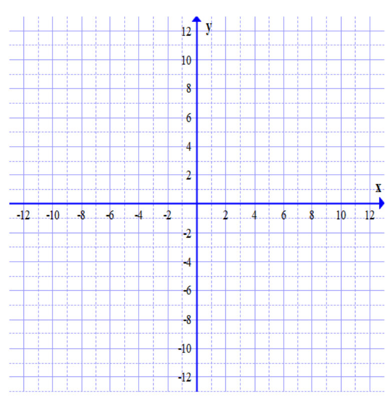

- One-to-one Property for Exponential Function (Relating the Bases)

  If $b^u = b^v$ then $u = v$

  This happens because exponential functions are one-to-one functions and by definition different x values have different y values and therefore if the y values are the same the x values are the same.

- Solving Exponential Equations: Relating the Bases or One-to-One Property
  - Method of Relating the Bases in Symbols

    $$b^u = b^v$$
    $$u = v$$

  - Method of Relating the Bases in Words
    - Requires the exponential equation to have the bases on both sides the same
    - When the bases are the same the exponents must be equal because of the one-to-one property of exponential functions.
  - Examples of Solving Exponential Equations by Relating the Bases
    - Example 1: Solve. $3^x = 81$
    - Example 2: Solve. $\sqrt{5} = 125^x$
    - Example 3: Solve. $\frac{1}{\sqrt[5]{216}} = 6^x$

▼ Example 4: Solve. $e^x = \frac{1}{e^{13}}$

▼ Example 5: Solve. $5^{x-1} = \frac{1}{25}$

▼ Example 6: Solve. $3^{x+1} = 9^{5x}$

▼ Example 7: Solve. $\left(e^{x^2}\right)^5 = e^{180}$

▼ Example 8: Solve. $6^{x^3} = \frac{1296^x}{216^{-x^2}}$

▼ Example 9: Solve. $e^{x^2} = e^x \cdot e^{12}$

▼ Example 10: Solve. $e^{x^2} = \frac{e^8}{(e^x)^2}$