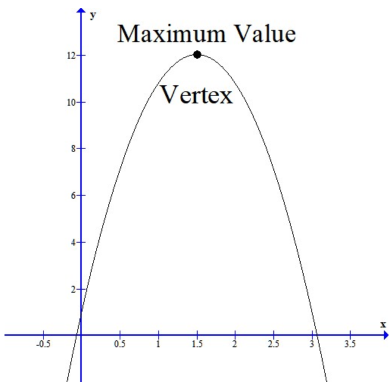
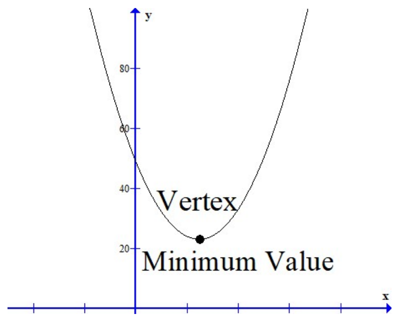
4.2 Applications of Quadratic Functions (More Space)

- Finding the minimum or maximum value of a quadratic function





Use the vertex formula to find the vertex. The vertex is labeled (h, k)

When the quadratic function is in general form $[f(x) = ax^2 + bx + c]$, the vertex can be found using the vertex formula. $h = -\frac{b}{2a}$ and $k = f(h)$.

The minimum or maximum value is the y coordinate (k) of the vertex.

- The Position Function: $s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$

The position of a free-falling object (neglecting air resistance) under the influence of gravity can be represented by the equation

$$s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$$

where s_0 is the initial height of the object, v_0 is the initial velocity of the object, and g is the acceleration due to gravity. On Earth, the value of g is approximately 32 feet per second per second or 9.8 meters per second per second.

  - Examples using the Position Function
    - Example 1

A person standing close to the edge on the top of a 96 foot building throws a baseball vertically upward. The quadratic function $s(t) = -16t^2 + 16t + 96$ models the ball's height above the ground, $s(t)$, in feet, t seconds after it was thrown.

a. After how many seconds does the ball reach its maximum height? What is the maximum height?

b. How many seconds does it take until the ball finally hits the ground?

c. Find, $s(0)$ and describe what this means.

▼ Example 2

A projectile is fired from a cliff 180 feet above the water at an inclination of 45 degrees to the horizontal, with a muzzle velocity of 45 feet per second. The height h of the projectile above the water is given by $h(x) = \frac{-32x^2}{45^2} + x + 180,$ where x is the horizontal distance of the projectile from the face of the cliff. Use this information to answer the following.

a. At what horizontal distance from the face of the cliff is the height of the projectile a maximum.

b. Find the maximum height of the projectile.

▼ Profit: $P = R - C$

P-profit
R-revenue
C-cost

▼ Example 1

The monthly revenue R achieved by selling x baseball gloves is figured to be $R(x) = 80x - 0.5x^2$. The monthly cost C of selling x baseball gloves is $C(x) = 20x + 1000$.

a. How many baseball gloves must the company sell to maximize revenue? What is the maximum revenue?

b. Profit is given as, $P(x) = R(x) - C(x)$. What is the profit function?

c. How many baseball gloves must the company sell to maximize profit? What is the maximum profit?

▼ Revenue: $R = xp$

R-revenue
x-number of units
p-price

▼ Example 2

The marketing department at an electronics company has found that, certain tablets are sold at a price of p dollars per unit, the number x of tablets sold is given by the demand equation $x = 35000 - 100p$

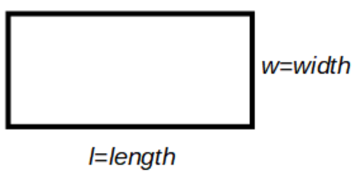
a. Using the formula $R = xp$, find the model that represents the revenue R as a function of the price p.

b. What unit price should be used to maximize revenue?

c. If this price is charged, what is the maximum revenue?

d. How many units are sold at this price?

▼ Area of a Rectangle: $A = lw$



$A = lw$
A-area
l-length
w-width

$P = 2l + 2w$
P-Perimeter
l-length
w-width

▼ Examples using Area

▼ Example 1

You have 80 yards of fencing to enclose a rectangular region. Find the dimensions of the rectangle that maximize the enclosed area. What is the maximum area?

▼ Example 2

Suppose you are asked to build a rectangular fence which boarders a river but you only have 1584 feet of fencing available. No fencing is required along the river. What should the length and width of the fence be in order to maximize the area?