3.5B Composition of Functions

▼ Build Composed Functions

▼ Notion

$(f \circ g)(x) = f(g(x))$

$(g \circ f)(x) = g(f(x))$

- This operation can be thought of as substituting one function inside another function.
- The function listed after the composition symbol is substituted into the function before the composition symbol.

▼ Examples: Building a composed function

Given $f(x) = x^2 - 2x - 3$ and $g(x) = x^2 - 9$, Find the following.

▼ Example 1: $(f \circ g)(x)$

▼ Example 2: $(g \circ f)(x)$

Given $f(x) = \frac{1}{x-3}$ and $g(x) = \sqrt{x+4}$, find the following

- Example 3: $(f \circ g)(x)$

- Example 4: $(g \circ f)(x)$

- Evaluate a Composed Function with the Equation
  - Build then plug
    - Example 1: Given $f(x) = x^2 - 2x - 3$ and $g(x) = x^2 - 9$

      Find the following $(f \circ g)(2)$

    - Example 2: Given $f(x) = \frac{1}{x-3}$ and $g(x) = \sqrt{x+4}$

      Find the following $(g \circ f)(4)$

▼ Plug then build (follow the symbols)

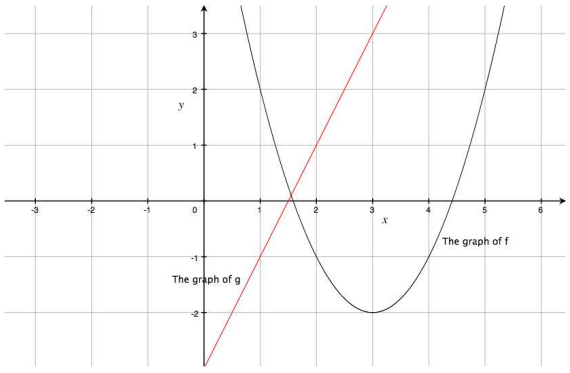
▼ Example 1: Given $f(x) = \frac{1}{x-3}$ and $g(x) = \sqrt{x+4}$

Find the following $(f \circ g)(0)$

▼ Example 2: Given $f(x) = x^2 - 2x - 3$ and $g(x) = x^2 - 9$

Find the following $(g \circ f)(-1)$

▼ Example: Evaluate a Composed Function with a Graph



$(f \circ g)(2)$

$(f \circ g)(3)$

$(g \circ f)(5)$

$(g \circ f)(1)$

▼ Find the Domain of Composed Functions

The domain of $(f \circ g)(x) = f(g(x))$ can be found by

1. Start with the domain of g because g is the inside function.
2. Adjust the domain of g by excluding any values of x where $g(x)$ is not in the domain of f.

The domain of $(g \circ f)(x) = g(f(x))$ can be found by

1. Start with the domain of f because f is the inside function.
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▼ Examples

▼ Given $f(x) = x^2 - 2x - 3$ and $g(x) = x^2 - 9$

▼ Find the domain of the given functions

$f(x) = x^2 - 2x - 3$ and $g(x) = x^2 - 9$

▼ Example 1: Find the domain of $(f \circ g)(x) = x^4 - 20x^2 + 96$

▼ Example 2: Find the domain of $(g \circ f)(x) = x^4 - 4x^3 - 2x^2 + 12x$

▼ Given $f(x) = \frac{1}{x-3}$ and $g(x) = \sqrt{x+4}$

▼ Find the domain of the given functions

$f(x) = \frac{1}{x-3}$ and $g(x) = \sqrt{x+4}$

▼ Example 3: Find the domain of $(f \circ g)(x) = \frac{1}{\sqrt{x+4}-3} = \frac{\sqrt{x+4}+3}{x-5}$

▼ Example 4: Find the domain of $(g \circ f)(x) = \sqrt{\frac{1}{x-3} + 4} = \frac{\sqrt{(4x+11)(x-3)}}{x-3}$