3.5A The Algebra of Functions

- ▼ Build Combined Functions
  - ▼ Notion

    $(f+g)(x) = f(x) + g(x)$

    $(f-g)(x) = f(x) - g(x)$

    $(fg)(x) = f(x)g(x)$

    $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
  - ▼ Examples
    - ▼ Given $f(x) = x^2 - 2x - 3$ and $g(x) = x^2 - 9$
      - ▼ Example 1: $(f+g)(x)$
      - ▼ Example 2: $(f-g)(x)$
      - ▼ Example 3: $(fg)(x)$
      - ▼ Example 4: $\left(\frac{f}{g}\right)(x)$

- Given $f(x) = \frac{1}{x-3}$ and $g(x) = \sqrt{x+4}$
  - Example 5: $(f+g)(x)$
  - Example 6: $(f-g)(x)$
  - Example 7: $(fg)(x)$
  - Example 8: $\left(\frac{f}{g}\right)(x)$

- Evaluate a Combined Function with the Equation
  - Given $f(x) = x^2 - 2x - 3$ and $g(x) = x^2 - 9$ (Build then plug)
    - Example 1: $(f+g)(2)$
      - From earlier

        $$(f+g)(x) = 2x^2 - 2x - 12$$

- Example 2: $(f-g)(-1)$

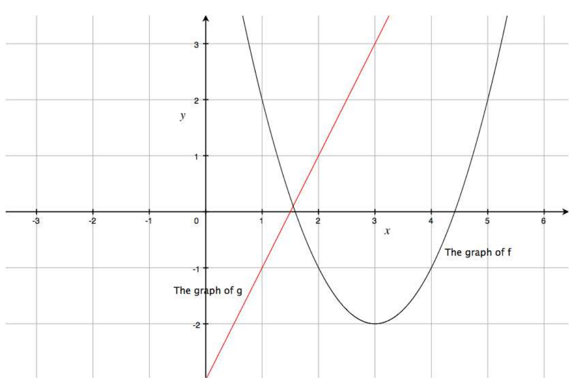
    - From earlier

      $(f-g)(x)=-2x+6$

- Given $f(x)=\frac{1}{x-3}$ and $g(x)=\sqrt{x+4}$ (Plug then build or follow the symbols)
    - Example 1: $(fg)(0)$

    - Example 2: $\left(\frac{f}{g}\right)(5)$

- Example: Evaluate a Combined Function with a Graph



$(f+g)(1)$

$(f-g)(2)$

$(fg)(3)$

$\left(\frac{f}{g}\right)(1)$

- Find the Domain of Combined Functions
  - The domain of $(f+g)(x)$ is the domain of f intersected with the domain of g. Symbolically, $D_{f+g} = D_f \cap D_g$
  - The domain of $(f-g)(x)$ is the domain of f intersected with the domain of g. Symbolically, $D_{f-g} = D_f \cap D_g$
  - The domain of $(fg)(x)$ is the domain of f intersected with the domain of g. Symbolically, $D_{fg} = D_f \cap D_g$
  - The domain of $\left(\frac{f}{g}\right)(x)$ is the domain of f intersected with the domain of g and $g(x) \neq 0$. Symbolically, $D_{\frac{f}{g}} = D_f \cap D_g$ and $g(x) \neq 0$
  - Examples
    - Given $f(x) = x^2 - 2x - 3$ and $g(x) = x^2 - 9$
      - Find the Domain of the given functions

        $f(x) = x^2 - 2x - 3$ and $g(x) = x^2 - 9$
      - Example 1: Find the Domain of $(f+g)(x) = 2x^2 - 2x - 12$
      - Example 2: Find the Domain of $(f-g)(x) = -2x + 6$
      - Example 3: Find the Domain of $(fg)(x) = x^4 - 2x^3 - 12x^2 + 18x + 27$
      - Example 4: Find the Domain of $\left(\frac{f}{g}\right)(x) = \frac{x^2-2x-3}{x^2-9} = \frac{x+1}{x+3}$

- Given $f(x) = \frac{1}{x-3}$ and $g(x) = \sqrt{x+4}$
  - Find the domain of the given functions

    $f(x) = \frac{1}{x-3}$ and $g(x) = \sqrt{x+4}$

  - Example 5: Find the Domain of $(f+g)(x) = \frac{1}{x-3} + \sqrt{x+4}$

  - Example 6: Find the Domain of $(f-g)(x) = \frac{1}{x-3} - \sqrt{x+4}$

  - Example 7: Find the Domain of $(fg)(x) = \frac{\sqrt{x+4}}{x-3}$

  - Example 8: Find the Domain of $\left(\frac{f}{g}\right)(x) = \frac{1}{(x+3)\sqrt{x+4}}$

▼ Applications of Combined Functions

▼ Revenue

$$R = xp$$

R-revenue
x-number of units
p-price

▼ Example: Build and evaluate the revenue function

The price p and the quantity x sold of a small flat-screen television set obeys the demand equation $p = -.16x + 240$

a) Express the revenue R as a function x. Use the formula $R = xp$.

b) How much should be charged for the television set if there are 60 television sets in stock?

c) How much should be charged for the television set if there are 750 television sets in stock?

d) What is the revenue when there are 750 sets in stock?

▼ Profit

$$P = R - C$$

P-profit
R-revenue
C-cost

▼ Example: Build and evaluate the profit function

Rite-Cut riding lawnmowers obey the demand equation $p = -\frac{1}{20}x + 1010$. The cost of producing x lawnmowers is given by the function $C(x) = 130x + 7000$.

a) Express the revenue R as a function x. Use the formula $R = xp$.

b) Express the profit P as a function x. Use the formula $P = R - C$.

c) What is the revenue if 8800 units are sold?

d) What is the profit if 8800 units are sold?

e) What price should be charged if 8800 units are sold?

▼ Area of a Rectangle

$$A = lw$$

R-area
l-length
p-width

▼ Example: Build and evaluate the area function

A farmer has 1,556 feet of fencing available to enclose a rectangular area bordering a river. No fencing is required along the river. Let x represent the length of the side of the rectangular enclosure that is perpendicular to the river.

a) Create an expression that represents the side parallel to the river.

b) What is the length of the side parallel to the river if the side perpendicular to the river is 389?

c) Create a function, A(x), that describes the total area of the rectangular function of x, where x is the length of the rectangular enclosure that is perpendicular to the river.

d) What is the area of the rectangle when the side perpendicular to the river is 389?