3.1B Relations and Functions (Notation, Evaluation, Domain)

- Definition of Function

  A **function** is a relation such that for each element in the domain, there is exactly one corresponding element in the range.

- Function Notation $y = f(x)$
  - f is the name of the function.
  - $f(x)$ is y
  - The parenthesis don't mean multiplication. They are a placeholder for values that will replace the variable.
  - x is the variable used in this equation.
  - Examples: Using $f(x) = 3x^2 + 5x - 2$
    - Example 1: Find $f(2)$
    - Example 2: Find $f(-1)$
    - Example 3: Find $f(-x)$
    - Example 4: Find $f(x+h)$

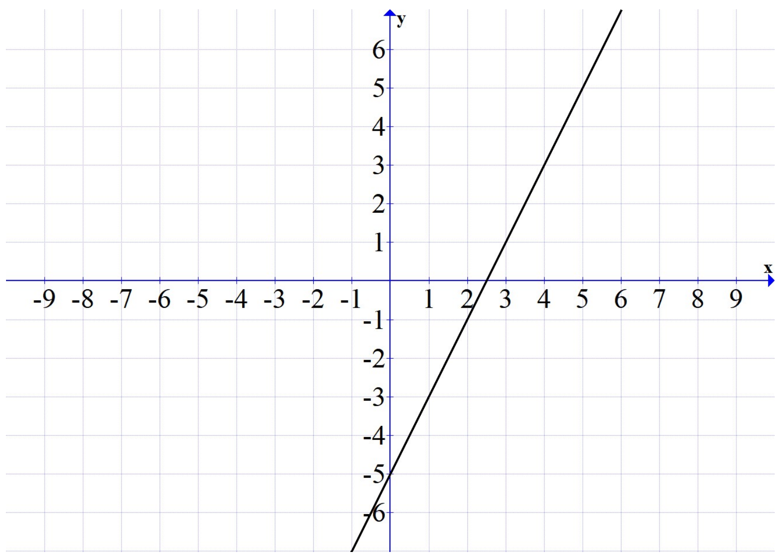
▼ Example 5: Find $f(x+h)-f(x)$

▼ Example 6: Find $\frac{f(x+h)-f(x)}{h}$

▼ The Difference Quotient

$$\frac{f(x+h)-f(x)}{h}$$

▼ Examples: Using a graph



- Find $f(1)$
- Find $f(2)$
- Find $f(3)$
- For what x is $f(x)=3$?
- For what x is $f(x)=-5$?
- For what x is $f(x)=5$?

- Finding the Domain of a Function Given it's Equation
  - Polynomial Functions
    - Definition of a Polynomial Function

      A polynomial function is of the form

      $$f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$$

      where n is a non-negative integer, such as $\{0, 1, 2, ...\}$ and the coefficients $\{a_n, a_{n-1}, ..., a_2, a_1, a_0\}$ are real numbers

    - Practical Identification of Polynomial Functions

      Look for the variable to be in the numerator. (If there is no fraction at all, the terms are in the numerator.) The variable should not be inside a radical or absolute value. The powers or exponents should be whole numbers. Whole numbers come from this list. $\{0, 1, 2, ...\}$

    - Finding Domain of Polynomial Functions

      There are no restrictions to the domain of a polynomial function. Meaning that each x in the real numbers will result in a y in the real numbers. For that reason the domain of all polynomials is all real numbers. In interval notation, that is $(-\infty, \infty)$.

    - Examples of Polynomial Functions

      $f(x) = 2x - 7$

      $g(x) = 2x^2 - 9x + 5$

      $h(x) = 7x^3 + 5x^7$

      $s(t) = 5$

  - Rational Functions
    - Definition of a Rational Function

      A rational function is of the form

      $$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomial functions and $q(x)$ is not equal to zero.

- Practical Identification of Rational Functions

  The numerator and denominator are polynomials. Most functions with variables in the denominator are considered rational functions but there are exceptions.Since rational functions are in the form of a fraction, there is a restriction that the denominator is not zero. You can find the restricted values be creating an equation. The equation is the denominator equal to zero.

- Finding Domain of Rational Functions

  Since rational functions are in the form of a fraction, there is a restriction that the denominator is not zero. You can find the restricted values be creating an equation. The equation is the denominator equal to zero.

  $$q(x) = 0$$

  The solutions to this equation are restricted values and must be excluded from the domain.

- Examples of Rational Functions

  $f(x) = \frac{2}{x-2}$

  $g(x) = \frac{x-5}{x^2-9}$

  $h(x) = \frac{2x}{6x^2-23x+20}$

  $s(t) = \frac{t^2+16}{t^3+5t^2+6t}$

- Radical\Root Functions (Even Index)
  - Definition of a Radical\Root Function (Even Index)

    A root function is of the form

    $$f(x) = \sqrt[n]{g(x)}$$

    where n is an even positive integer greater than or equal to 2.
  - Practical Identification of Radical\Root Functions (Even Index)

    The variable is inside or under the radical. The index of the radical is an even number $\{2, 4, 6, \ldots\}$. The square root is an even index although the index is not written.

    It is also possible to write radicals as fractional exponents.
  - Finding Domain of Radical\Root Functions (Even Index)

    Taking the even root of a negative number results in a complex or imaginary number. Since we are interested in real function values we would like the expression inside the radical to be non-negative (zero or positive). Create an inequality to solve. The inequality is the expression under the radical greater than or equal to zero.

    $g(x) \geq 0$

    The solution set of this inequality is the domain.
  - Examples of Radical\Root Functions (Even Index)

    $f(x) = \sqrt{x-5}$

    $g(x) = \sqrt[4]{6-3x}$

$h(x) = \sqrt[6]{x^2 - 3x + 5}$

$s(t) = (2t - 3)^{1/2}$

- Radical\Root Functions (Odd Index)
  - Definition of a Radical\Root Function (Odd Index)

    A root function is of the form

    $$f(x) = \sqrt[n]{g(x)}$$

    where n is an odd positive integer greater than or equal to 2.
  - Practical Identification of Radical\Root Functions (Odd Index)

    The variable is inside or under the radical. The index of the radical is an odd number. $\{3, 5, 7, ...\}$ The cube root is an odd index.

    It is also possible to write radicals as fractional exponents.
  - Finding Domain of Radical\Root Functions (Odd Index)

    There are no restrictions to the domain of root function with odd index. For that reason the domain of all root function with odd index is all real numbers. In interval notation, that is $(-\infty, \infty)$
  - Examples of Radical\Root Functions (Odd Index)

    $f(x) = \sqrt[3]{x - 5}$

    $g(x) = \sqrt[5]{6 - 3x}$

    $h(x) = \sqrt[7]{x}$

    $s(t) = (2t - 3)^{1/3}$
- Extra Practice

http://math15fun.com/wp-content/uploads/2020/05/Finding-Domain.pdf

http://math15fun.com/wp-content/uploads/2021/01/Finding-Domain-1-of-2-Solutions.pdf