3.1A Relations and Functions (Function Definition)

- Definition of Relation

  A **relation** is a set of ordered pairs.

- Definition of Domain

  The **domain** is the set of all first coordinates. (x's)

- Definition of Range

  The **range** is the set of all second coordinates. (y's)

- Definition of Function

  A **function** is a relation such that for each element in the domain, there is exactly one corresponding element in the range.

- Examples: Is the set of ordered pairs a function? Find the domain and range.
  - Example 1:

    $\{(2,3),(1,9),(-2,8),(5,2)\}$

  - Example 2:

    $\{(-2,3),(5,6),(-2,1),(3,8)\}$

  - Example 3:

    $\{(1,10),(2,10),(3,10)\}$

- Examples: Is the equation a function?
  - Example 1: Is the equation a function?

    $4x - 5y = 20$

  - Example 2: Is the equation a function?

    $y = 2x^2 - 3x + 1$

  - Example 3: Is the equation a function?

    $x = y^2$

  - Example 4: Is the equation a function?

    $(x+1)^2 + y^2 = 9$

  - Example 5: Is the equation a function?

    $y = |2x - 3|$

  - Example 6: Is the equation a function?

    $x = |y|$

- Example Takeaways
    - Solve for y or write equations "y=" form to help determine if the equations is a function. (If you end up with $\pm$ this indicates two y's for every x and the equation is not a function)
    - Even powers on y lead to $\pm$ when you solve for y. Therefore equations with an even power on y are not functions.
    - Absolute value on y lead to $\pm$ when you solve for y. Therefore equations with an absolute value on y are not functions.

- Vertical Line Test
    - What is it used for?

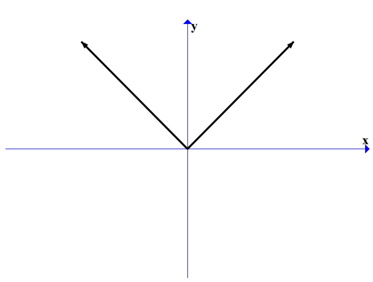
      The vertical line test is used to determine if a graph represents a function.
    - Why use a vertical line?

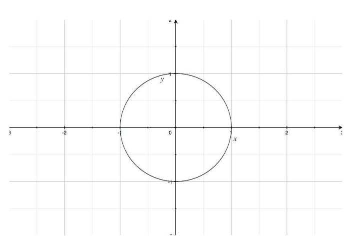
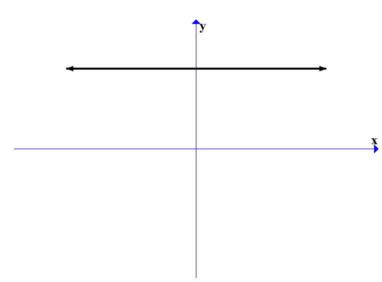
      A vertical line violates the definition of a function since it has one x-value and infinite y values
    - What is it?

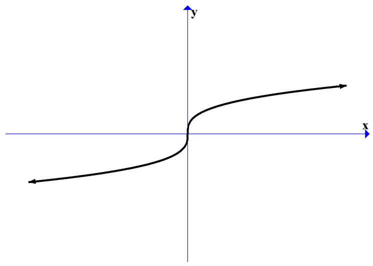
      A graph in the Cartesian plane is the graph of a function if and only if no vertical lines intersects the graph more than once.
    - How to use it?

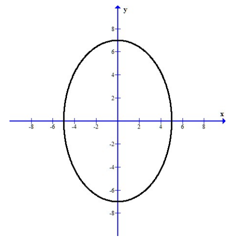
      Draw vertical lines on the graph. Count the number of intersection points. If there is more than one intersection point on any vertical line, the graph is not a function. If every vertical line has one or no intersection points then the graph represents a function.

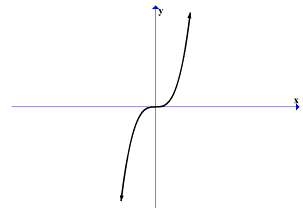
- Examples: Does the graph represent a function?

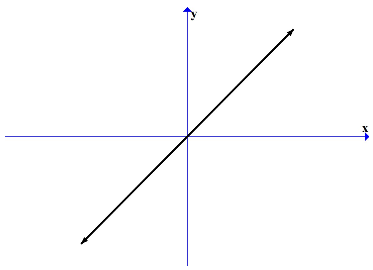


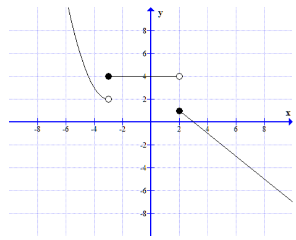



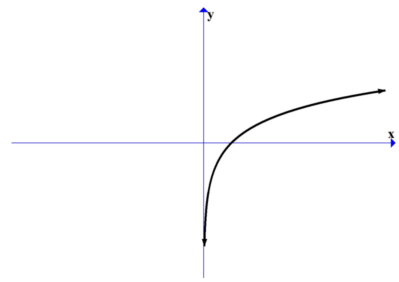


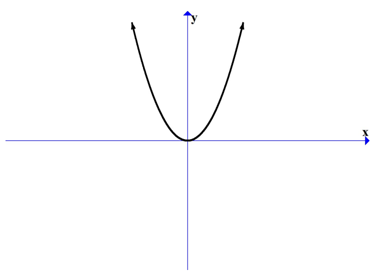


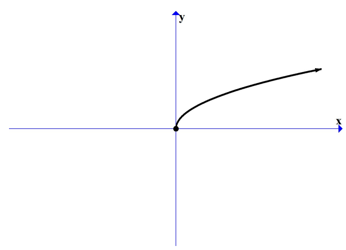


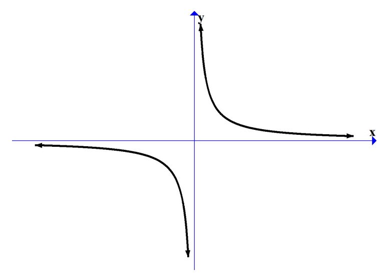


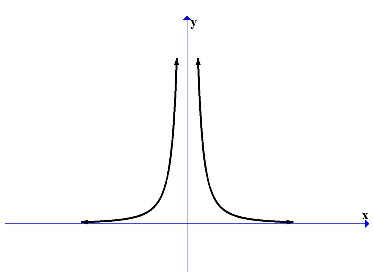


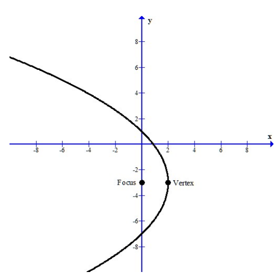


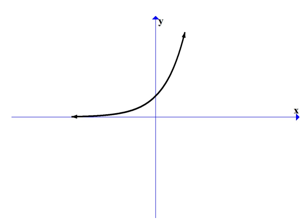












- ▼ Classifying Functions
  - ▼ Polynomial Functions
    - ▼ Definition of a Polynomial Function

      A polynomial function is of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$$

where n is a non-negative integer, such as $\{0, 1, 2, ...\}$ and the coefficients $\{a_n, a_{n-1}, ..., a_2, a_1, a_0\}$ are real numbers

▼ Practical Identification of Polynomial Functions

Look for the variable to be in the numerator. (If there is no fraction at all, the terms are in the numerator.) The variable should not be inside a radical or absolute value. The powers or exponents should be whole numbers. Whole numbers come from this list. $\{0, 1, 2, ...\}$

▼ Examples of Polynomial Functions

$f(x) = 2x - 7$

$g(x) = 2x^2 - 9x + 5$

$h(x) = 7x^3 + 5x^7$

$s(t) = 5$

▼ Rational Functions

▼ Definition of a Rational Function

A rational function is of the form

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomial functions and $q(x)$ is not equal to zero.

▼ Practical Identification of Rational Functions

The numerator and denominator are polynomials. Most functions with variables in the denominator are considered rational functions but there are exceptions. Having a negative in the exponent means reciprocal of the base and can also indicate a rational function.

▼ Examples of Rational Functions

$f(x) = \frac{2}{x-2}$

$g(x) = \frac{x-5}{x^2-9}$

$h(x) = 2x^{-4}$

$s(t) = \frac{t^2+16}{t^3+5t^2+6t}$

- ▼ Radical\Root Functions (Even Index)
  - ▼ Definition of a Radical\Root Function (Even Index)

    A root function is of the form

    $$f(x) = \sqrt[n]{g(x)}$$

    where n is an even positive integer greater than or equal to 2.
  - ▼ Practical Identification of Radical\Root Functions (Even Index)

    The variable is inside or under the radical. The index of the radical is an even number $\{2, 4, 6, ...\}$. The square root is an even index although the index is not written.

    It is also possible to write radicals with fractional exponents.
  - ▼ Examples of Radical\Root Functions (Even Index)

    $f(x) = \sqrt{x-5}$

    $g(x) = \sqrt[4]{6-3x}$

    $h(x) = \sqrt[6]{x^2-3x+5}$

    $s(t) = (2t-3)^{1/2}$
- ▼ Radical\Root Functions (Odd Index)
  - ▼ Definition of a Radical\Root Function (Odd Index)

    A root function is of the form

    $$f(x) = \sqrt[n]{g(x)}$$

    where n is an odd positive integer greater than or equal to 2.
  - ▼ Practical Identification of Radical\Root Functions (Odd Index)

The variable is inside or under the radical. The index of the radical is an odd number. $\{3, 5, 7, ...\}$ The cube root is an odd index.

It is also possible to write radicals as fractional exponents.

▼ Examples of Radical\Root Functions (Odd Index)

$f(x) = \sqrt[3]{x-5}$

$g(x) = \sqrt[5]{6-3x}$

$h(x) = \sqrt[7]{x}$

$s(t) = (2t-3)^{1/3}$