3.6B Inverse Functions

▼ Definition of Inverse Function

Let f be a one-to-one function with domain A and range B. Then f^{-1} is the inverse of f with domain B and range A. Furthermore, if $f(a) = b$ then $f^{-1}(b) = a$.

▼ Find the inverse from a set of ordered pairs, find the domain and range, determine if each set of ordered pairs is a one-to-one function.

   ▼ Example 1: $\{(1, 10), (2, 10), (3, 10)\}$

   ▼ Example 2: $\{(2, 3), (1, 9), (-2, 8), (5, 2)\}$

   ▼ Example 3: $\{(-2, 3), (5, 6), (-2, 1), (3, 8)\}$

   ▼ Example Takeaways

   - Switch the x and y to find the inverse

   - The inverse is a function only when the original function is one-to-one

   - The domain of f is the range of f^{-1}

   - The range of f is the domain of f^{-1}

▼ Verify the Functions are Inverse Functions

    ▼ Cancellation Properties of Inverse Functions

$$(f \circ f^{-1})(x) = x$$
$$(f^{-1} \circ f)(x) = x$$

Note: This property is true for every x using the definition of an inverse function. From the definition of an inverse function $f(a) = b$ then $f^{-1}(b) = a$.

$(f^{-1} \circ f)(a) = f^{-1}(f(a)) = f^{-1}(b) = a$ and $(f \circ f^{-1})(b) = f(f^{-1}(b)) = f(a) = b$

▼ Determine whether f and g are inverse functions by evaluating $(f \circ g)(x)$ and $(g \circ f)(x)$.

    ▼ Example 1: $f(x) = \frac{3}{2}x - 7$ and $g(x) = \frac{2x-14}{3}$

    ▼ Example 2: $f(x) = \frac{5-x}{x}$ and $g(x) = \frac{5}{x+1}$

▼ Process of finding inverses from an equation

    1. Change $f(x)$ to y.

    2. Switch the x and y.

    3. Solve for y.

▼ Examples: Find the inverse of the given one-to-one function. Verify the functions are inverses by calculating $f \circ f^{-1}$ and $f^{-1} \circ f$. Find the domain and range of the function and it's inverse.
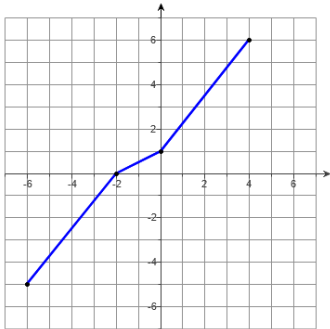
▼ Example 1: $f(x) = 2x - 6$

▼ Example 2: $g(x) = x^3 + 1$

▼ Example 3: $h(x) = \frac{5}{x} + 4$

▼ Example 4: $r(x) = -x^2 + 6, \, x \geq 0$

▼ Answer questions about the f^{-1} using the graph of f

a) What is the domain of f^{-1}?

b) What is the range of f^{-1}?

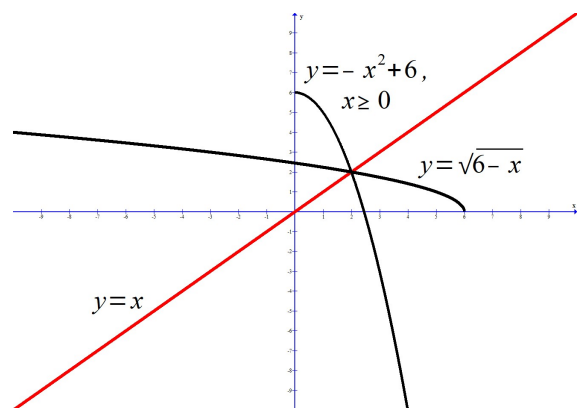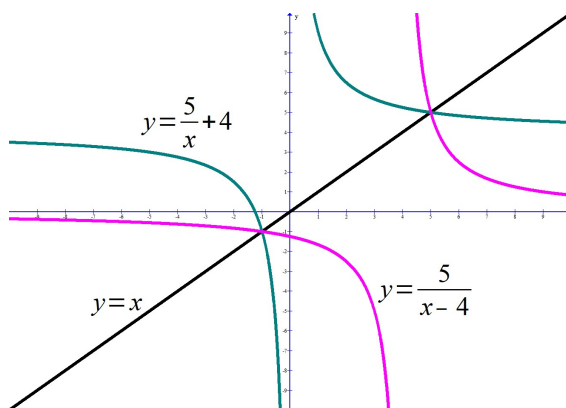c) What is the y-intercept of f^{-1}?
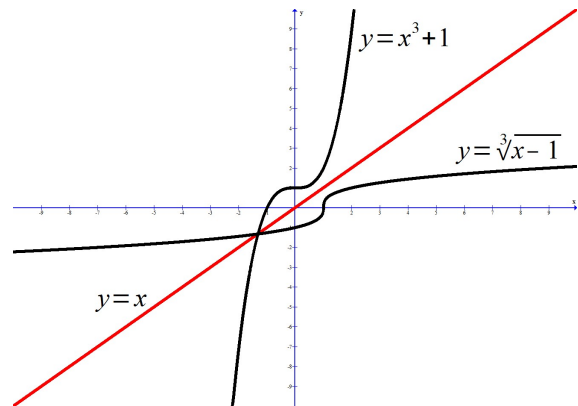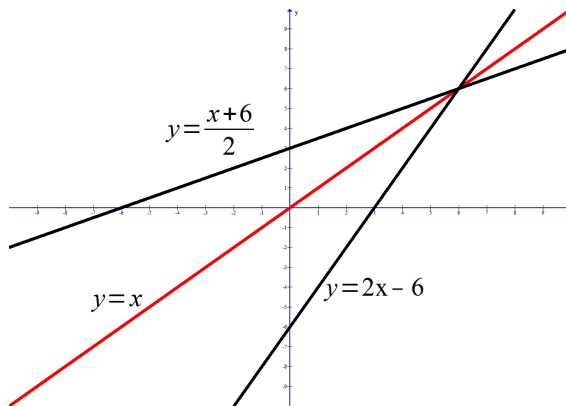
d) Evaluate $f^{-1}(0)$.

e) Evaluate $f^{-1}(-5)$.

f) Evaluate $f^{-1}(6)$.

g) Evaluate $f^{-1}(1)$.

▼ Symmetry and Inverse Functions

A function and it's inverse are symmetric around the line $y = x$.

$y=\dfrac{x+6}{2}$

$y=x$

$y=2x-6$

$y=x^3+1$

$y=\sqrt[3]{x-1}$

$y=x$

$y=\dfrac{5}{x}+4$

$y=x$

$y=\dfrac{5}{x-4}$

$y=-x^2+6,\ x\ge 0$

$y=\sqrt{6-x}$

$y=x$

▼ Use the graph of f to sketch a graph of f^{-1}.