9.4 Mathematical Induction

▼ The Principle of Mathematical Induction

Suppose that the following two statements are satisfied with regard to a statement about the natural numbers:

Condition 1: The statement is true for the natural number one

Condition 2: If the statement is true for some natural number k, it is also true for the next natural number k+1.

Then the statement is true for all natural numbers.

▼ Two ways to prove condition 2

1. Start with one side of the statement you want to prove and work your way to the other side.

2. Start with the statement you assume and work your way to the statement you want to prove

▼ Example 1: Show that the following statement is true for all natural numbers

$$1 + 3 + 5 + ... + (2n - 1) = n^2$$

▼ Example 2: Show that the following statement is true for all natural numbers

$$1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$$

▼ Example 3: Show that the following statement is true for all natural numbers

$$1 + 3 + 3^2 + ... + 3^{n-1} = \frac{1}{2}(3^n - 1)$$