9.3 Geometric Sequence; Geometric Series

- Definition of a **Geometric Sequence**

  When the ratio between successive terms of a sequence is constant, the sequence is called geometric. The constant is called the common ratio.

- Examples: Is the sequence geometric? If so, find the first term and the common ratio.
  - Example 1: $\{3, 6, 12, 24, ...\}$
  - Example 2: $\{4, 6, 8, 10, ...\}$
  - Example 3: $\{a_n\} = \{2^n\}$
  - Example 4: $\{t_n\} = \{2 \cdot 3^{n-1}\}$
  - Example 5: $\{s_n\} = \{4^{n/3}\}$
  - Example 6: $b_1 = 1; b_n = nb_{n-1}$
  - Example 7: $c_1 = 8; c_n = 3c_{n-1}$

- Formula for the nth term of an geometric sequence

  $a_n = a_1 \cdot r^{n-1}$

  a_1 is the first term in the sequence

  r is the common ratio

- Formula for the recursive definition of an geometric sequence

  $a_1 = a; a_n = ra_{n-1}$

  a is the first term

  r is the common ratio

- Examples:

  Write the nth term of the geometric sequence and the recursive definition.

  - Example 1: $\{3, 6, 12, 24, ...\}$
  - Example 2: $\{4, 12, 36, 108, ...\}$
  - Example 3: Find the 8th term of the geometric sequence. Then find the recursive definition.

    $\{8, 4, 2, 1, ...\}$

  - Example 4: Find the nth term of the geometric sequence.

    $r = -3; a_6 = -243$

- Formula for sum of the first n terms of a geometric sequence

$$S_n = a_1 + a_1 r + a_1 r^2 + ... + a_1 r^{n-1}$$
$$= \sum_{i=1}^{n} a_1 r^{i-1} = a_1 \cdot \frac{1 - r^n}{1 - r}$$

- Formula for infinite sum of a geometric sequence

$$\sum_{i=1}^{\infty} a_1 r^{i-1} = \frac{a_1}{1 - r} \qquad \text{if } |r| < 1$$

- Examples: Find the sum of the geometric sequence
  - Example 1: Find the sum of the first n terms of the sequence

    $\frac{1}{7}+\frac{6}{7}+\frac{6^2}{7}+\frac{6^3}{7}+...+\frac{6^{n-1}}{7}$

  - Example 2: Find the sum of the first n terms of the sequence

    $$\sum_{i=1}^{n}\left(\frac{3}{8}\right)^{i}$$

  - Example 3: Solve the application problem related to sum of a geometric sequence.

    Consider a game board that has 10x10=100 squares. You put one grain of sand on the first square of the game board, two grains of sand on the second square, four grains of sand on the third square, continuing until you have filled the board. Compute the total number of grains needed to do this.