8.7 Systems of Inequalities

▼ What is an **inequality**?

An **inequality** is like an equation but instead of an equal symbol ($=$) an inequality symbol ($<, >, \leq, \geq$) is used to relate expressions.

▼ How do you solve an inequality with two variables?

Solutions to inequalities in two variables are **order pairs**. In most cases, there are infinite solutions and these solutions will be represented by drawing a graph of the solution set.

The graph will consist of a boundary line or curve and a shaded region.

1. **Boundary**: When the inequality symbol is $<$ (less than) or $>$ (greater than) the ordered pairs on the boundary line are not part of the solution set. The boundary line will be drawn dashed to indicate this. When the inequality symbol $\leq$ (less than or equal to) or $\geq$ (greater than or equal to) the ordered pairs on the boundary line are part of the solution set. The boundary line will be drawn solid to indicate that the boundary is included. Replace the inequality symbol with an equal symbol. Draw the line or curve with a dashed or solid line.

2. **Shading**: The boundary divides the plane into regions. The whole region will contain ordered pairs that are either solutions to the inequality or not. Choose a test point that is not on the boundary line. Substitute the test point into the original inequality to determine if it is a solution or not. If it is a true it is a solution and all of the order pairs in that region are solutions. If it is false it is not a solution and all of the ordered pairs in that region are not solutions. We use shading to indicate the regions that are solutions. If the inequality is true shade that region that contains the test point. If the inequality is false shade the region that does not contain the test point.
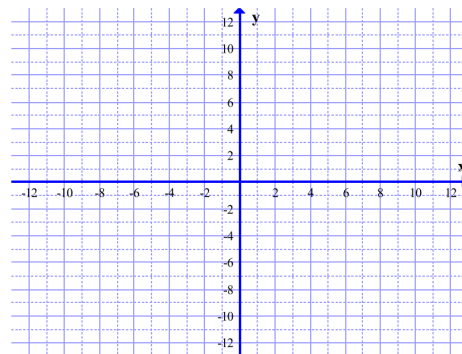
▼ Examples of linear inequalities

▼ Example 1: Find the solutions to the inequality by graphing the inequality.

$$2x - 3y \geq 6$$
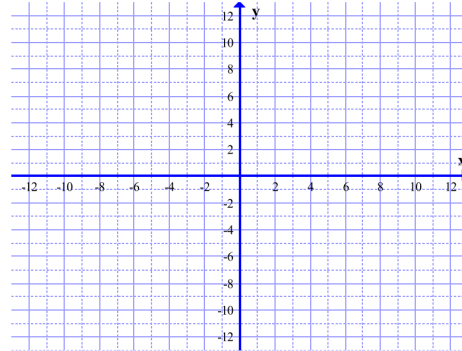
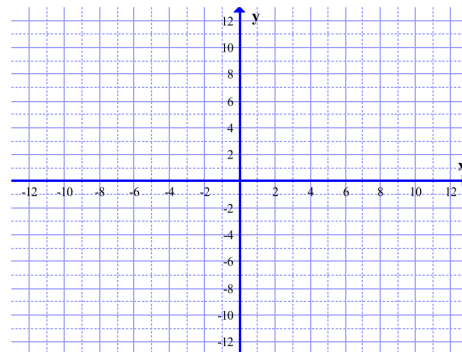▼ Example 2: Find the solutions to the inequality by graphing the inequality.

$$y > -\tfrac{2}{3}x$$
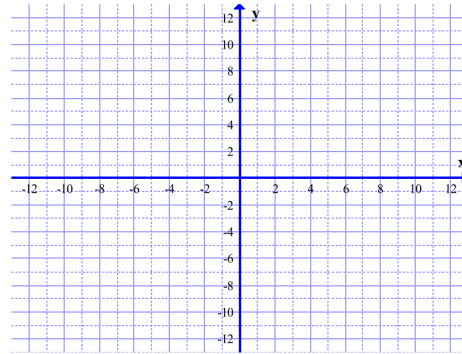
▼ Examples of non-linear inequalities

▼ Example 1: Find the solutions to the inequality by graphing the inequality.

$$x^2 + y^2 \leq 9$$

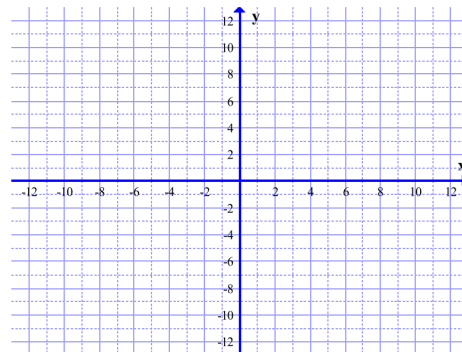▼ Example 2: Find the solutions to the inequality by graphing the inequality.

$$y < x^2 + 2$$

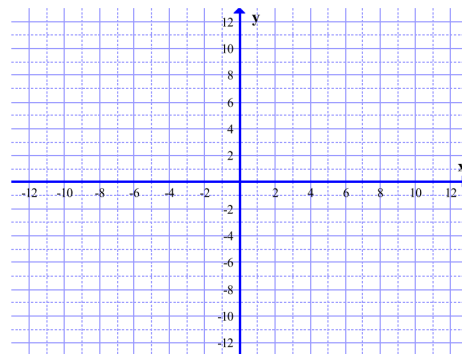▼ Examples of systems of linear inequalities

▼ Example 1: Find the solutions to the system by graphing the inequalities in the same plane.

$$\begin{cases} x > 5 \\ y \leq 4 \end{cases}$$

▼ Example 2: Find the solutions to the system by graphing the inequalities in the same plane.
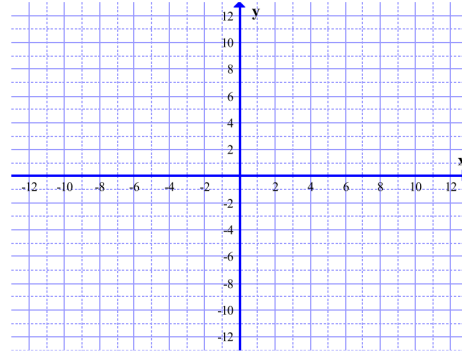
$$\begin{cases} y \leq \frac{5}{2}x - 3 \\ 2x + 5y > 10 \end{cases}$$

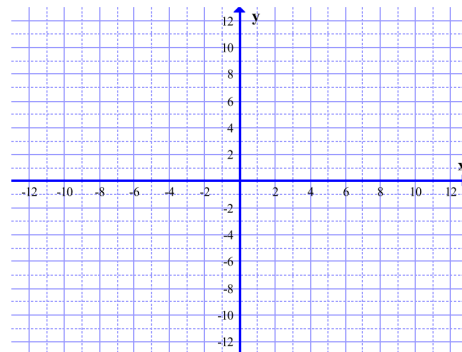▼ Examples of systems of non-linear inequalities

▼ Example 1: Find the solutions to the system by graphing the inequalities in the same plane.

$$\begin{cases} x^2 + y^2 \leq 36 \\ x - y \leq 6 \end{cases}$$

▼ Example 2: Graph the solution set.

$$\begin{cases} y > x^2 - 8 \\ y \leq x - 1 \end{cases}$$

▼ Example 3: Graph the solution set.

$$\begin{cases} xy \geq 8 \\ y \geq x^2 + 2 \end{cases}$$