8.5 Partial Fraction Decomposition

▼ Setting the stage for partial fraction decomposition

Write as a single fraction: $\frac{2}{x-3} + \frac{5}{x+4}$

Decompose $\frac{P}{Q}$

▼ Case 1: *Q* has only nonrepeated linear factors

Assuming that Q has only nonrepeated linear factors, the polynomial Q has the form $Q(x) = (x - a_1)(x - a_2) \cdot ... \cdot (x - a_n)$, where no two of the numbers $a_1, a_2, ..., a_n$ are equal. In this case, the partial fraction decomposition of $\frac{P}{Q}$ is of the form

$$\frac{P(x)}{Q(x)} = \frac{A_1}{x - a_1} + \frac{A_2}{x - a_2} + ... + \frac{A_n}{x - a_n}$$

where the numbers $A_1, A_2, ..., A_n$ are to be determined.

▼ Example 1: Find the partial fraction decomposition.

$\frac{x}{x^2-5x+6}$

▼ Case 2: *Q* has repeated linear factors

If the polynomial Q has a repeated linear factor, say $(x-a)^n$, $n \geq 2$ an integer, then, in the partial fraction decomposition of $\frac{P}{Q}$, allow for the terms

$$\frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + ... + \frac{A_n}{(x-a)^n}$$

where the numbers $A_1, A_2, ..., A_n$ are to be determined.

▼ Example 2: Find the partial fraction decomposition

$\frac{x+2}{x^3-2x^2+x}$

▼ Example 3: Find the partial fraction decomposition

$\frac{x^3-8}{x^2(x-1)^2}$

▼ Case 3: *Q* contains a nonrepeated irreducible quadratic factor

If Q contains a nonrepeated irreducible quadratic factor of the form $ax^2 + bx + c$, then, in the partial fraction decomposition of $\frac{P}{Q}$, allow for the term

$$\frac{Ax + B}{ax^2 + bx + c}$$

where the numbers A and B are to be determined.

▼ Example 4: Find the partial fraction decomposition

$\frac{3x-5}{x^3-1}$

▼ Case 4: *Q* contains a repeated irreducible quadratic factor

If the polynomial Q contains a repeated irreducible quadratic factor $(ax^2 + bx + c)^n$, $n \geq 2$, n an integer, then, in the partial fraction decomposition of $\frac{P}{Q}$, allow for terms

$$\frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + ... + \frac{A_nx + B_n}{(ax^2 + bx + c)^n}$$

▼ Example 5: Find the partial fraction decomposition

$\frac{x^3+x^2}{(x^2+4)^2}$

▼ Summary

- Improper fractions: Use long division if the degree of the numerator is larger than the denominator
- Write the equation following cases 1-4
- Multiply both sides by the original denominator to clear fractions
- Solve for missing coefficients by “equating coefficients” or choosing strategic values of x
- Write the expression as a decomposed fraction

▼ Extra Examples

▼ Example 6: Find the partial fraction decomposition

$$\frac{x^3+x^2-3}{x^2+3x-4}$$

▼ Example 7: Find the partial fraction decomposition

$$\frac{x^4-5x^2+x-4}{x^2+4x+4}$$