8.4 Matrix Algebra

- Adding and Subtracting Matrices
  - Matrices must be the same size (row and column)
  - Addition and subtraction is done entry by entry
  - Example of Adding and Subtracting Matrices

    $A = \begin{bmatrix} 2 & 4 & 8 & -3 \\ 0 & 1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 4 & 0 & 1 \\ 6 & 8 & 2 & 0 \end{bmatrix}$

    a) Find $A + B$ b) Find $A - B$

- Commutative Property of Matrix Addition

  $A + B = B + A$

- Associative Property of Matrix Addition

  $(A + B) + C = A + (B + C)$

- Definition: Zero Matrix
  - matrix of any size where all entries are zero.

  $A + 0 = 0 + A$

- Scalar Multiplication

  $A = \begin{bmatrix} 3 & 1 & 5 \\ -2 & 0 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 & 0 \\ 8 & 1 & -3 \end{bmatrix}$ and $C = \begin{bmatrix} 9 & 0 \\ -3 & 6 \end{bmatrix}$

  a) Find $4C$ b) Find $3A - 2B$

- Matrix Multiplication
  - Let matrix A be a matrix with m rows and n columns
  - Let matrix B be a matrix with s rows and t columns
  - For the product AB to be defined, the number of columns of A must equal the number of rows of B $(n = s)$.
  - The product AB is a matrix with m rows and t columns.
  - Example: Multiply the Matrices by hand and with a calculator

    $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$

    a) AB

    b) BA

- Definition: Identity Matrix

  A matrix with the same number of rows and columns (square matrix) that has 1's as entries along the diagonal and 0's as the other entries is called an identity matrix.

  This identity matrix is usually notated as I_n.

- Identity Property

  If A is an n by n square matrix, $AI_n = I_nA = A$.

- Example: Multiply the Matrices by hand and with a calculator

  $A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 4 & 6 \\ 5 & 2 \end{bmatrix}$

  a) AI_3 b) I_2A c) BI_2

- Definition: Inverse Matrix

  Let A be a square n by n matrix. If there exist an n by n matrix A^{-1} (read as "A inverse") for which $AA^{-1} = A^{-1}A = I_n$ then A^{-1} is called the inverse of matrix A.

- Multiply a Matrix by Its Inverse

  $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$

- Finding the Inverse of a Matrix by hand and with a calculator

  $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

▼ Show the matrix has no inverse by hand and with a calculator

$$A = \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$$

▼ Finding the Inverse of a Matrix by hand and with a calculator

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix}$$

▼ Use the Inverse Matrix to Solve a System of Linear Equations

$$\begin{cases} x + y = 3 \\ -x + 3y + 4z = -3 \\ 4y + 3z = 2 \end{cases}$$