8.3 Systems of Linear Equations: Determinants

▼ Definition of 2 by 2 Determinant

If a, b, c, and d are four real numbers, the symbol

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

is called a 2 by 2 determinant. Its value is the number $ad - bc$.

▼ Example: Find the determinant of a 2 by 2 Matrix

$$A = \begin{bmatrix} 3 & -2 \\ 6 & 1 \end{bmatrix}$$

▼ Cramer's Rule for Two Equations Containing Two Variables

The solution to the system of equations

$$\begin{cases} ax + by = s \\ cx + dy = t \end{cases}$$

is given by

$$x = \frac{\begin{vmatrix} s & b \\ t & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{D_x}{D} \qquad y = \frac{\begin{vmatrix} a & s \\ c & t \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{D_y}{D}$$

provided that

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \neq 0$$

- Example: Solve the System using Determinants

$$\begin{cases} 3x - 2y = 4 \\ 6x + y = 13 \end{cases}$$

- Example: Find the determinant of a 3 by 3 Matrix

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 4 & 6 & 2 \\ 8 & -2 & 3 \end{bmatrix}$$

- Cramer's Rule for Three Equations Containing Three Variables

$$x = \frac{D_x}{D} \quad y = \frac{D_y}{D} \quad z = \frac{D_z}{D}$$

- Example: Solve the System using Determinants

$$\begin{cases} 2x + y - z = 3 \\ -x + 2y + 4z = -3 \\ x - 2y - 3z = 4 \end{cases}$$