### 7.3 The Ellipse

- Geometric Definition of an Ellipse

An ellipse is a collection of all points in the plane, the sum of whose distances from two fixed points called the foci is a constant.


Vocabulary of an Ellipse


V Important Components of the Equation
$a$ : distance from center to vertex
$b$ : distance from center to minor vertex

$c$ : distance from center to focus
Related by Pythagorean theorem.

## Labeling the Ordered Pairs and Equations

When you know the center, $a, b$ and $c$ you can find all of the significant features of the ellipse.


Equations of an ellipse with center at the origin $(0,0)$.

Major Axis is Horizontal $(y=0)$

$$
\begin{gathered}
\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1 \\
b^{2}+c^{2}=a^{2}
\end{gathered}
$$

Major Axis is Vertical $(x=0)$

$$
\begin{aligned}
& \frac{x^{2}}{b^{2}}+\frac{y^{2}}{a^{2}}=1 \\
& b^{2}+c^{2}=a^{2}
\end{aligned}
$$

V Equations of an ellipse with center at $(h, k)$.

Major Axis is Horizontal $(y=k)$
$\frac{(x-h)^{2}}{a^{2}}+\frac{(y-k)^{2}}{b^{2}}=1$;
$b^{2}+c^{2}=a^{2}$

Major Axis is Vertical $(x=h)$

$$
\begin{aligned}
& \frac{(x-h)^{2}}{b^{2}}+\frac{(y-k)^{2}}{a^{2}}=1 \\
& b^{2}+c^{2}=a^{2}
\end{aligned}
$$

$\nabla$ Examples: Given the graph find the features and equation

## V Example 1:

Find the equation of the ellipse, the center, vertices, foci, major axis, minor axis


## V Example 2:

Find the equation of the ellipse, the center, vertices, foci, major axis, minor axis

$\boldsymbol{V}$ Examples: Given the features find the equation and the graph

## - Example 1:

Find the orientation, center, vertices, and write the equation of the ellipse.
Graph the Ellipse
The foci is $(0, \pm 2)$ and the length of the major axis is 8 .


## V Example 2:

Find the orientation, center, vertices, and write the equation of the ellipse.

The center is at $(2,1)$; One vertex is at $(7,1)$; One focus is at $(-1,1)$.

$\boldsymbol{\nabla}$ Examples: Given the equations find the features and graph

## - Example 1:

Find the orientation, center, vertices, and graph of the ellipse.

$$
\frac{(x-3)^{2}}{16}+\frac{(y+4)^{2}}{49}=1
$$

## V Example 2:

Rewrite using completing the square.
Find the orientation, center, vertices, and graph of the ellipse.

$$
x^{2}-2 x+16 y^{2}+32 y+1=0
$$



