# 6.6A Exponential Equations

▼ Solving Exponential Equations

▼ Method: Relating the Bases or One-to-One Property

▼ Method of Relating the Bases in Symbols and Words

1. Write the equation in a form where the bases are the same on both sides. $[b^u = b^v]$
2. Use the one-to-one property which states if the bases are the same then the exponents are equal. $[u = v]$

▼ Examples of Solving Exponential Equations by Relating the Bases

▼ Example 1: $4^{2x+1} = 8^x$

▼ Example 2: $e^{-x^2} = (e^x)^2 \cdot \frac{1}{e^3}$

▼ Example 3: $3^{7x} = 9^{2x-5}$

▼ Example 4: $3^{5x} \cdot 9^{x^2} = 27$

▼ Example 5: $5^x - 2 = 23$

▼ Example 6: $3^{x+1} = 9^{5x}$

▼ Example 7: $7^{x^2+3x} = \frac{1}{49}$

▼ Example 8: $9^{x^2} = 3^{-5x-2}$

▼ Example 9: $5^{2x^2+3x} = 25^{6-x}$

▼ Example 10: $\left(e^{-x}\right)^2 = \frac{e^x}{e^2}$

▼ Method: Convert to Logarithmic Equation

▼ Method of Converting to a Logarithm in Symbols and Words

1. Recognize an exponential equation. $y = b^x$
2. Rewrite the exponential equation as a logarithmic equation using the following equivalence. $[\log_b y = x]$
3. Solve the remaining equation to find solutions.

▼ Examples of Solving Exponential Equations

▼ Example 1: $2^x = 5$

▼ Example 2: $8 \cdot 3^x = 5$

▼ Example 3: $5^{x-2} = 3^{3x+2}$

▼ Example 4: $3^x = 8$

▼ Example 5: $10^x = 1500$

▼ Example 6: $5^{x-3} = 137$

▼ Example: 7: $7^{2x+1} = 3^{x+5}$

▼ Example 8: $500e^{3x} + 40 = 1040$

▼ Method: Take the Log of Both Sides

▼ Method of Take the Log of Both Sides in Symbols and Words

1. Recognize an exponential equation.
2. Use the one-to-one property for logarithms and take the natural log of both sides.
3. Use the power rule for logarithms which allows you to move the power inside the logarithm to become the coefficient of the logarithm.
4. Solve for the variable by dividing.

$$
\begin{aligned}
y &= b^x \\
\ln y &= \ln b^x \\
\ln y &= x \ln b \\
\frac{\ln y}{\ln b} &= \frac{x \ln b}{\ln b} \\
\frac{\ln y}{\ln b} &= x
\end{aligned}
$$

▼ Examples of Solving Exponential Equations

▼ Example 1: $2^x = 5$

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- Example 3: $5^{x-2} = 3^{3x+2}$
- Example 4: $3^x = 8$
- Example 5: $10^x = 1500$
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- Method: Quadratic in Form or u-substitution
  - Method of Quadratic in Form in Symbols and Words
    1. Recognize an equation is quadratic in form $[a \cdot w^{2x} + b \cdot w^x + c = 0]$
    2. The substitution $u = w^x$ turns the equation into a quadratic equation. $[au^2 + bu + c = 0]$
    3. Solve the quadratic equation by factoring, the square root method, completing the square or the quadratic formula. $[u = l$ or $u = m]$

4. Return to the original variable using the same substitution as before. $[w^x = l$ or $w^x = m]$

5. Solve the remaining exponential equation.

▼ Examples of Solving Exponential Equations by u-substitution

▼ Example 1: $e^{2x} - 2e^x - 3 = 0$

▼ Example 2: $4^x - 2^x - 12 = 0$