6.2A One-to-one Functions

▼ Definition of One-to-One Function

A function f is **one-to-one** if for any values $a \neq b$ in the domain of f, $f(a) \neq f(b)$

▼ Examples to determine if a set of order pairs represent a one-to-one function

   ▼ Example 1: $\{(1, 10), (2, 10), (3, 10)\}$

   ▼ Example 2: $\{(2, 3), (1, 9), (-2, 8), (5, 2)\}$

   ▼ Example 3: $\{(-2, 3), (5, 6), (-2, 1), (3, 8)\}$

▼ The Horizontal Line Test

If every horizontal line intersects the graph of a function f at most once, then f is one-to-one.

   ▼ What is it used for?

   The horizontal line test is used to determine if a graph of a function represents a one-to-one function.

   ▼ Why use a horizontal line?

   A horizontal line violates the definition of a one-to-one function since every x has the same y value. One-to-one functions don't have repeated y values.

   ▼ What is it?

   A graph of a function in the Cartesian plane is the graph of a one-to-one function if and only if no horizontal lines intersects the graph more than once.

   ▼ How to use it?

Draw horizontal lines on the graph of the function. Count the number of intersection points. If there is more than one intersection point (more than one x with the same y) on any horizontal line, the graph of the function is not a one-to-one function. If every horizontal line has one or no intersection points (every x has a different y) then the graph of the function represents a one-to-one function.
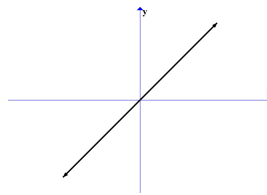
▼ Examples

    ▼ Determine whether the basic functions are one-to-one functions.

Constant:

$$f(x) = c$$
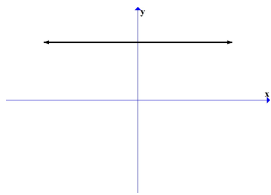
Identity:
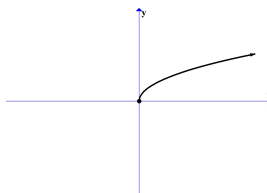
$$f(x) = x$$

Square:

$$f(x) = x^2$$
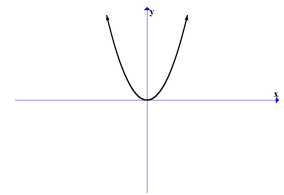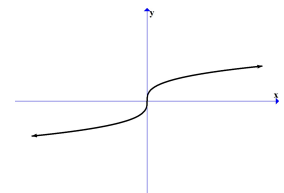
Cube:

$$f(x) = x^3$$
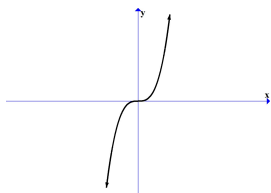
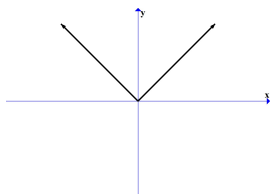Square Root:

$$f(x) = \sqrt{x}$$

Cube Root:

$$f(x) = \sqrt[3]{x}$$
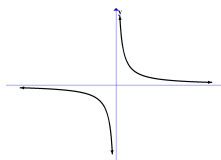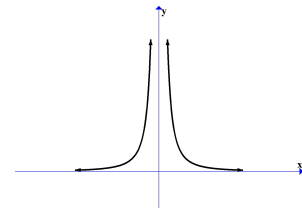
Absolute Value:

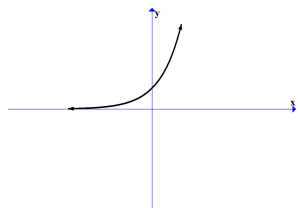$$f(x) = |x|$$

Reciprocal:

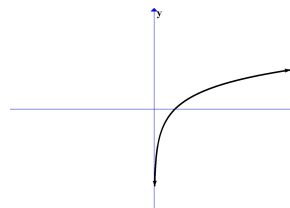$$f(x) = \frac{1}{x}$$

Reciprocal Squared:

$$f(x) = \frac{1}{x^2}$$

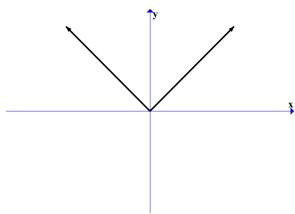Exponential Base e: $f(x) = e^x$ Logarithmic Base e: $f(x) = \ln x$
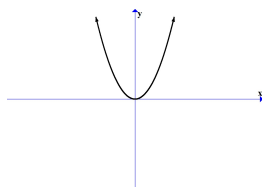
▼ Restricting the Domain

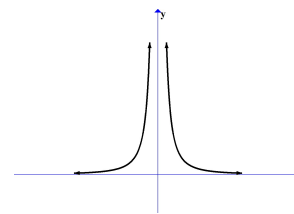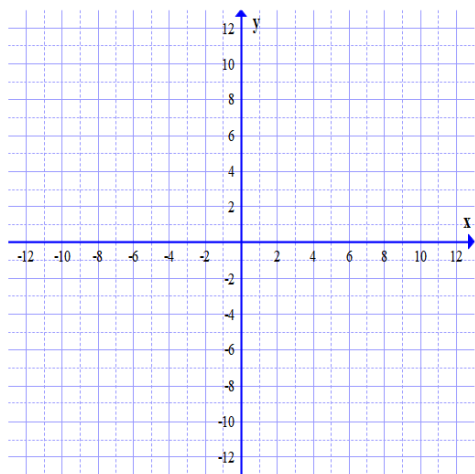▼ $f(x) = |x|$ ▼ $f(x) = x^2$ ▼ $f(x) = \frac{1}{x^2}$

▼ Piecewise Functions: Draw the graph & determine if the function is one-to-one

▼ Example 1:   ▼ Example 2:

$$f(x) = \begin{cases} x + 5 & x < 2 \\ -x - 3 & x \geq 2 \end{cases}$$ $$f(x) = \begin{cases} x^2 & x \leq 0 \\ -x - 5 & x > 0 \end{cases}$$