5.3 Complex Zeros; Fundamental Theorem of Algebra

▼ Definition: Complex Variable

A variable in the complex number system is referred to as a **complex variable**.

▼ Definition: Complex Polynomial

A **complex polynomial** function f of degree n is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$$

where the coefficients are complex numbers, n is a non negative integer, and x is a complex variable.

▼ Definition: Complex Zero

A complex number r is called a **complex zero** of f if $f(r) = 0$.

▼ The Fundamental Theorem of Algebra

- Every complex polynomial function f with degree greater than or equal to 1 has at least 1 complex zero.
- That is, every complex polynomial function of degree n greater than or equal to 1 has exactly n complex zeros, some of which may repeat.

▼ The Conjugate Pairs Theorem

Let $f(x)$ be a polynomial function whose coefficients are real numbers. If $r = a + bi$ is a zero of f, the complex conjugate $a - bi$ is also a zero of f.

▼ Example: Let f be a polynomial function of degree 5 with real coefficients. f has known zeros of 1, $5i$, and $1 + i$.

a) Find the remaining zeros with the conjugate pair theorem and the fundamental theorem of algebra

b) Write the equation in expanded form.

▼ Example: $h(x) = 6x^5 + 6x^4 + 66x^3 + 66x^2 - 480x - 480$

Given that $-4i$ is a zero of the function, find the remaining zeros of the function.

▼ Example: $f(x) = 3x^4 + 5x^3 + 25x^2 + 45x - 18$

a) Find the zeros of the complex polynomial.

b) Write the complex polynomial in factored form.