5.2 The Real Zeros of a Polynomial Function

▼ Remainder Theorem

Let f be a polynomial function. If f(x) is divided by x-c, then the remainder is f(c).

▼ Refresher Example: Find the reminder when $f(x) = x^3 - 4x^2 - 5$ is dividing by $x - 3$ with long division.

▼ Refresher Example: Find the reminder when $f(x) = x^3 - 4x^2 - 5$ is dividing by $x + 2$ with synthetic division

▼ Example: Find the reminder when $f(x) = x^3 - 4x^2 - 5$ is dividing by $x - 3$ with the remainder theorem

▼ Example: Find the reminder when $f(x) = x^3 - 4x^2 - 5$ is dividing by $x + 2$ with the remainder theorem

▼ Factor Theorem

Let f be a polynomial function, then $x - c$ is a factor of f if and only if $f(c) = 0$.

1) If $f(c) = 0$ then $x - c$ is a factor of f.

2) If $x - c$ is a factor of f then $f(c) = 0$.

▼ Example: Use the factor theorem and remainder theorem to determine whether the function $f(x) = 2x^3 - x^2 + 2x - 3$ has a factor of $x - 1.$ If $x - 1$ is a factor, write the polynomial in factored form.

▼ Example: Use the factor theorem and remainder theorem to determine whether the function $f(x) = 2x^3 - x^2 + 2x - 3$ has a factor of $x + 2.$ If $x + 2$ is a factor, write the polynomial in factored form.

▼ Theorem: Number of Real Zeros

A polynomial function cannot have more real zeros than its degree.

▼ Rational Zeros Theorem

Let f be a polynomial function of degree 1 or higher of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$$

where each coefficient is an integer, and $a_n \neq 0$. If $\frac{p}{q}$ is in lowest terms and is a rational zero of f, then p must be a factor of a_0 and q must be a factor of a_n.

▼ Example: For the function $f(x) = 2x^3 + 11x^2 - 7x - 6$, use the rational zeros theorem to list the potential zeros.

▼ Example: Find the zeros of $f(x) = 2x^3 + 11x^2 - 7x - 6$.

▼ Example: Write the function $f(x) = 2x^3 + 11x^2 - 7x - 6$ in factored form.

▼ Definition: Irreducible

In general, a quadratic factor is **irreducible** if it cannot be factored over the real numbers, that is, it is prime over the real numbers.

Examples: $x^2 + 1$; $x^2 - 6x + 13$

▼ Theorem: Factoring Polynomials

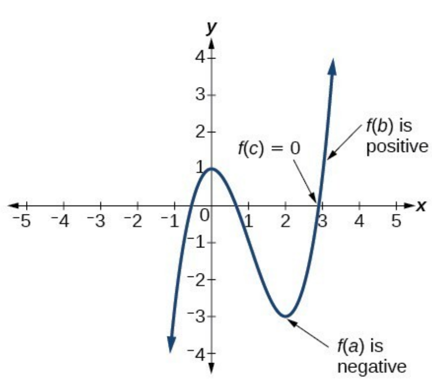
Every polynomial function (with real coefficients) can be uniquely factored into a product of linear factors and/or irreducible quadratic factors.

▼ Corollary: At least One Real Zero

A polynomial function (with real coefficients) of odd degree has at least one real zero.

▼ Intermediate Value Theorem

Let f denote a continuous function. If $a < b$ and if $f(a)$ and $f(b)$ are of opposite signs, then f has at least one zero between a and b.



▼ Example: Use the Intermediate Value Theorem to show that $f(x) = 3x^3 - 10x + 9$ has a zero in the interval $[-3, -2]$. Approximate the zero correct to two decimal places.