5.1 Polynomial Functions and Models

▼ Definition of a Polynomial Function

A polynomial function is of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$$

where n is a non-negative integer, such as $\{0, 1, 2, ...\}$ and the coefficients $\{a_n, a_{n-1}, ..., a_2, a_1, a_0\}$ are real numbers. n is called the **degree** of the polynomial. $a_n x^n$ is the **leading term** and a_0 is the **constant term**.

▼ Practical Identification of Polynomial Functions

Look for the variable to be in the numerator. (If there is no fraction at all, the terms are in the numerator.) The variable should not be inside a radical or absolute value. The powers or exponents should be whole numbers. Whole numbers come from this list. $\{0, 1, 2, ...\}$

▼ Identify polynomial functions

▼ Functions that are Polynomials

$f(x) = 2x - 7$

$g(x) = 2x^2 - 9x + 5$

$h(x) = 7x^3 - 28x^7$

$s(t) = 5$

$f(x) = 2x(x-3)(x+7)$

$g(x) = 3x(x-8)^2(x+2)^3$

$h(x) = -2(x-4)^3(x+1)^2$

$s(t) = \frac{1}{2}t - \sqrt{3}$

▼ Functions that are not Polynomials

$f(x) = \frac{3x-4}{x^2-9}$

$g(x) = x^{-1} = \frac{1}{x}$

$h(x) = \sqrt{2x-9}$

$s(t) = (2-t)^{1/2} = \sqrt{2-t}$

$f(x) = |7x - 2|$

$g(x) = e^x$

$h(x) = \log_3(x-7)$

$s(t) = \sin t$

▼ Definition Turning Points

Points on the graph that change from increasing to decreasing or decreasing to increasing are called **turning points.**

▼ Theorem: Turning Points

- If f is a polynomial function of degree n, then f has at most $n-1$ turning points.
- If the graph of of a polynomial function f has $n-1$ turning points, the degree of f is at least n.

▼ State the degree, maximum number of turning points, the leading term, and constant term

$f(x)=2x-7$ $\qquad$ $g(x)=2x^2-9x+5$ $\qquad$ $h(x)=7x^3-28x^7$

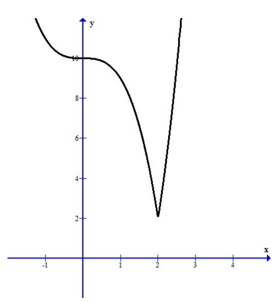
$s(t)=5$ $\qquad$ $f(x)=2x(x-3)(x+7)$ $\qquad$ $s(t)=\frac{1}{2}t-\sqrt{3}$

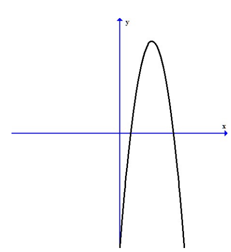
$h(x)=-2(x-4)^3(x+1)^2$ $\qquad$ $g(x)=3x(x-8)^2(x+2)^3$

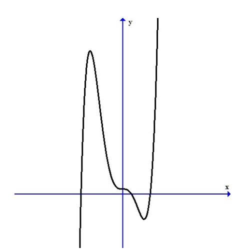
▼ The Graph of a Polynomial Function

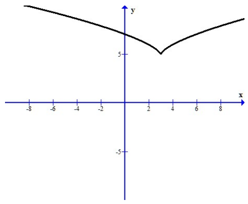
The graph of a polynomial function is both smooth and continuous. Smooth means that the graph will contain no sharp corners or cusps. Continuous means that the graph has no gaps or holes and can be drawn without lifting your pen from the paper.

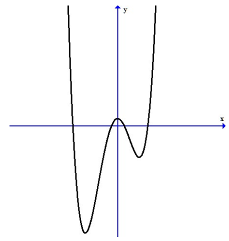
▼ Identify the graph of a polynomial function, state the number of turning points, and the least degree

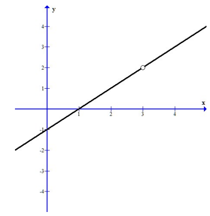


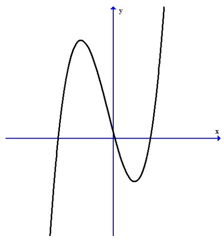


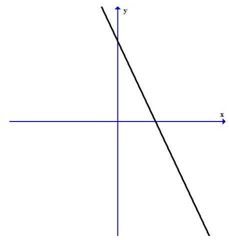


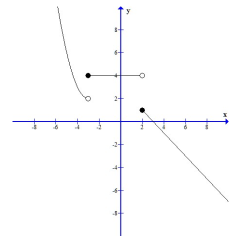






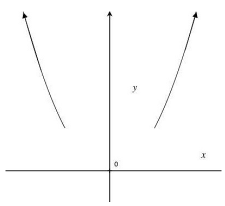
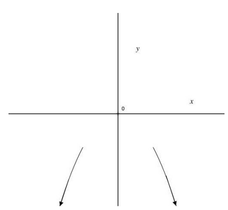
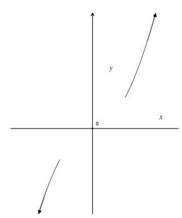
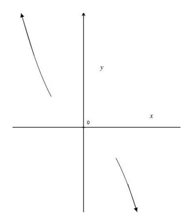






▼ End Behavior

For large values of x, either positive or negative, the graph of the polynomial function $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ resembles the graph of the power function $y = a_n x^n$.

Even degree, positive leading coefficient	Even degree, negative leading coefficient	Odd degree, positive leading coefficient	Odd degree, negative leading coefficient
			
$\lim\limits_{x \to -\infty} f(x) = \infty$	$\lim\limits_{x \to -\infty} f(x) = -\infty$	$\lim\limits_{x \to -\infty} f(x) = -\infty$	$\lim\limits_{x \to -\infty} f(x) = \infty$
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▼ Zeros of a polynomial

If f is a function and r is a real number for which $f(r) = 0$ then r is called a real zero of f.

Equivalent Statements

- r is a real zero of a polynomial function
- r is an x-intercept of the graph of f.
- $x - r$ is a factor of f.
- r is a solution to the equation $f(x) = 0$.

▼ Multiplicity of zeros

- If the factor of $x - r$ occurs more than once, r is called a repeated, or multiple zero or root of f.
- If $(x - r)^m$ is a factor of a polynomial f, then r is called a zero or root of multiplicity m.
- If r is a zero of **even** multiplicity

  ...the sign of $f(x)$ does not change from one side of r to the other.

  ...the graph graph **touches** the x-axis at r.
- If r is a zero of **odd** multiplicity...

  ...the sign of $f(x)$ changes from one side of r to the other.

  ...the graph graph **crosses** the x-axis at r.

▼ List the zeros and it's multiplicity, determine if the graph crosses or touches at the x-intercept, determine the end behavior

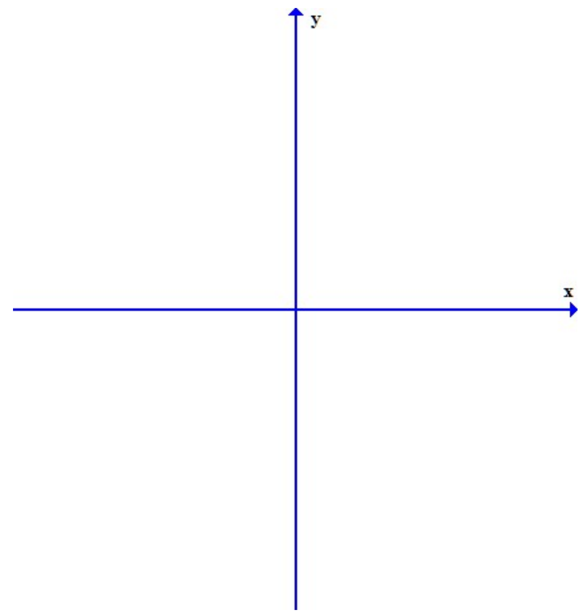
$f(x) = 2x - 7$ $\quad$ $g(x) = 2x^2 - 9x + 5$ $\quad$ $h(x) = 7x^3 - 28x^7$

$s(t) = 5$ $\quad$ $f(x) = 2x(x - 3)(x + 7)$ $\quad$ $s(t) = \frac{1}{2}t - \sqrt{3}$

$h(x) = -2(x - 4)^3(x + 1)^2$ $\quad$ $g(x) = 3x(x - 8)^2(x + 2)^3$

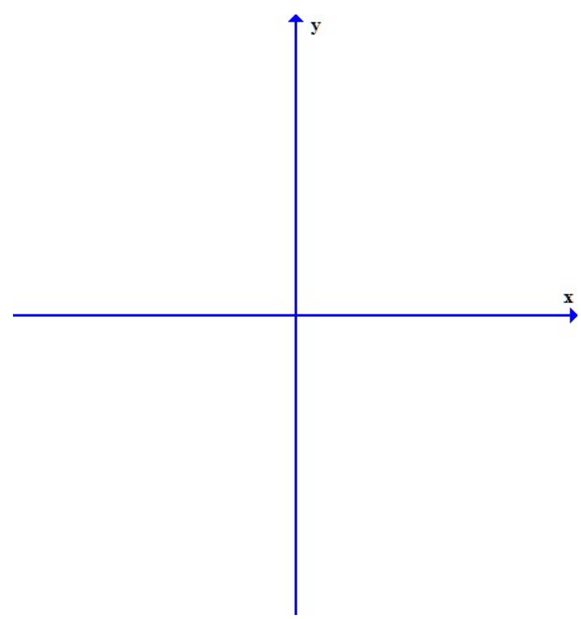
▼ Analyze the polynomial function $g(x) = 3x(x-8)^2(x+3)$

Graph, approximate the turning points, determine intervals of increasing and decreasing, domain and range



▼ Analyze the polynomial function $h(x) = 7x^3 - 28x^7$

Graph, approximate the turning points, determine intervals of increasing and decreasing, domain and range



▼ Analyze the polynomial function $f(x) = x^3 + 4x^2 - 4x - 16$

Find the zeros, end behavior, graph, approximate the turning points, determine intervals of increasing and decreasing, domain and range