

4.1 Linear Functions and Their Properties

▼ Linear Functions

Linear functions are written in the slope intercept form of a line. Slope intercept form is $y = mx + b$ where m is the slope of the line and $(0, b)$ is the y-intercept of the line. Written in function form we have $f(x) = mx + b$

▼ Theorem: Average Rate of Change of a Linear Function

Linear functions have a constant average rate of change. That is, the average rate of change of a linear function $f(x) = mx + b$ is $\frac{\Delta y}{\Delta x} = m$

Proof:

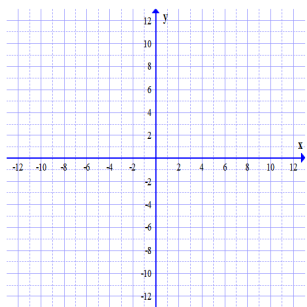
▼ Theorem: Increasing, Decreasing, and Constant Linear Functions

A linear function $f(x) = mx + b$ is increasing over the domain if its slope, m , is positive. It is decreasing over its domain if its slope, m , is negative. It is constant over its domain if its slope, m , is zero.

▼ Examples: Find the slope, y-intercept, graph, average rate of change. Determine if the linear function is increasing, decreasing or constant.

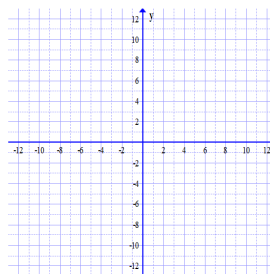
▼ Example 1:

$$f(x) = 3x - 5$$



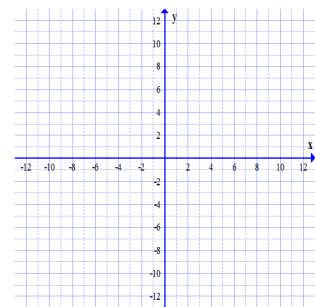
▼ Example 2:

$$f(x) = -\frac{2}{3}x + 7$$



▼ Example 3:

$$f(x) = 3$$



▼ Modeling with Linear Functions

If the average rate of change of the function is constant m , a linear function f can be used to model the relation between the two variables as follows: $f(x) = mx + b$ where b is the value of f at 0; that is, $b = f(0)$ or the initial value.

▼ Example of Modeling

Suppose that a company just purchased at the cost of \$28,000 per car the company depreciates the vehicles using the straight line method over 7 years. This means that each care will depreciate $\frac{28,000}{7} = 4,000$ each year.

a) Write a linear function that expresses the book value V of each car as a function of its age, x .

b) What is the book value after 3 years?

c) When will the book value be 8000?

▼ Business Vocabulary

The **quantity supplied** of a good is the amount of a product that a company is willing to make available for sale at a given price.

The **quantity demanded** of a good is the amount of a product that consumers are willing to purchase at a given price.

The **equilibrium price** of a product is defined as the price at with quantity supplied equals quantity demanded. $S = D$

The **equilibrium quantity** is the amount demanded (or supplied) a the equilibrium price.

The **revenue**, R , is the amount collected from selling goods.

The **cost**, C , is the amount it takes to produce the goods.

The **profit**, P , is the amount left after all costs are paid.

The **break-even point** is the point at which the company's profit equal zero ($P = 0$) or the revenue equals cost ($R = C$).

▼ Example of Supply and Demand

Suppose that the quantity supplied, S , and the quantity demanded, D , of cell phones each month are given by the following functions:

$$S(p) = 30p - 900 \text{ and } D(p) = -7.5p + 2850$$

where p is the price (in dollars) of the cell phone.

a) Find the equilibrium price of cell phones. What is the equilibrium quantity?

b) Determine the prices for which quantity supplied is greater than the quantity demanded.

▼ Example of Revenue, Cost, Profit

Suppose that a company revenue follows the model $R(x) = 8x$ and the cost follows the model $C(x) = 4.5x + 17,500$.

a) Find the company's break-even point.

b) Find the values of x such that the revenue is greater than cost.