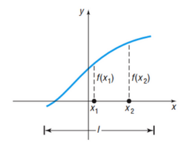
3.3 Properties of Functions

▼ Increasing, Decreasing, Constant

▼ Definition of Increasing

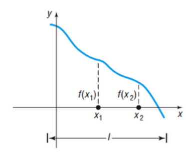
A function f is **increasing** on an interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$. (The graph of an increasing function always goes "up" from left to right.)



(a) For $x_1 < x_2$ in I, $f(x_1) < f(x_2)$; f is increasing on I.

▼ Definition of Decreasing

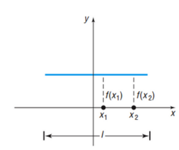
A function f is **decreasing** on an interval I if, for any choice of x_1 and x_2 in I with $x_1 < x_2$, we have $f(x_1) > f(x_2)$. (The graph of an decreasing function always goes "down" from left to right.)



(b) For $x_1 < x_2$ in I, $f(x_1) > f(x_2)$; f is decreasing on I.

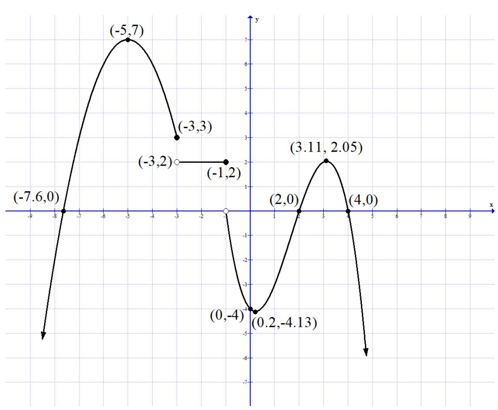
▼ Definition of Constant

A function f is constant on an interval I if, for all choices of x in I, the values of $f(x)$ are equal. (The graph of an constant function always stays the same from left to right.)



(c) For all x in I, the values of f are equal; f is constant on I.

▼ Find the Intervals of Increasing, Decreasing and Constant



Identify intervals of increasing

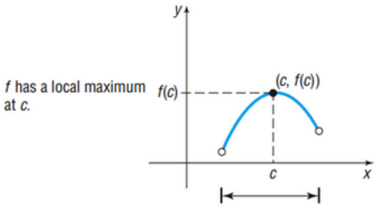
Identify intervals of decreasing

Identify intervals of constant

- Local Minimum or Local Maximum

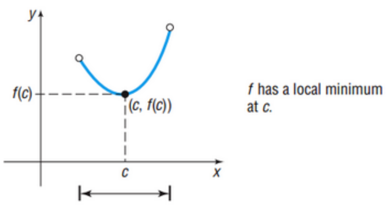
  - Definition of Local Maximum

    Let f be a function defined on some interval I. A function f has a **local maximum** at c if there is an open interval in I containing c so that, for all x in this open interval, we have $f(x) \leq f(c)$. We call $f(c)$ a local maximum value of f.

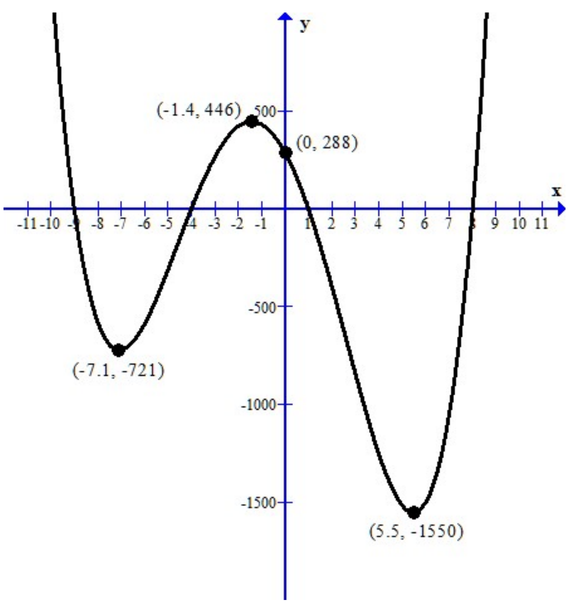


  - Definition of Local Minimum

    Let f be a function defined on some interval I. A function f has a **local minimum** at c if there is an open interval in I containing c so that, for all x in this open interval, we have $f(x) \geq f(c)$. We call $f(c)$ a local minimum value of f.



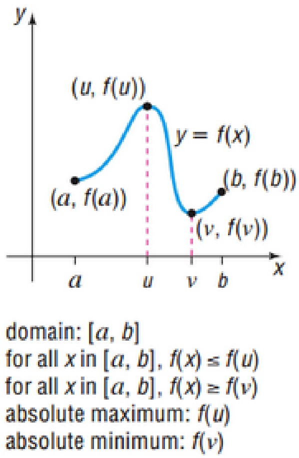
- Find the Local Minimum and Local Maximum



Identify any local minimums and where it occurs

Identify any local maximums and where it occurs

▼ Absolute Minimum or Local Maximum



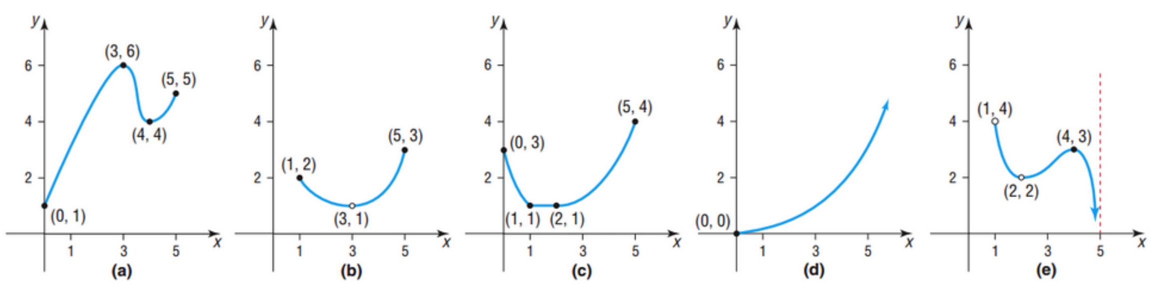
▼ Definition of Absolute Maximum

Let f be a function defined on some interval I. If there is a number u in I for which $f(x) \leq f(u)$ for all x in I, then f has an absolute maximum at u, and the number $f(u)$ is the absolute maximum of f on I.

▼ Definition of Absolute Minimum

Let f be a function defined on some interval I. If there is a number v in I for which $f(x) \geq f(v)$ for all x in I, then f has an absolute minimum at v, and the number $f(v)$ is the absolute minimum of f on I.

▼ Find the Absolute Min or Max if it Exists



▼ Continuous function

A precise definition requires calculus. A working definition is that a continuous functions is one whose graph has no gaps or holes and can be traced without lifting a pencil from the paper.

▼ Extreme Value Theorem

If f is a continuous function whose domain is a closed interval $[a, b]$, then f has an absolute maximum and an absolute minimum on $[a, b]$.

- Average Rate of Change

  If a and b, $a \neq b$, are in the domain of a function $y = f(x)$, the average rate of change of f from a to b is defined as

  Average rate of change $= \frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a} \quad a \neq b$

- Find the Average Rate of Change.
  - Example 1

    Find the average rate of change for $f(x) = x^2 - 9$ from 1 to 3.

    Find the average rate of change for $f(x) = x^2 - 9$ from 1 to 2.

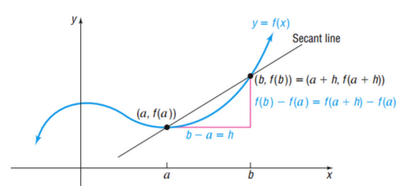
  - Example 2

    Find the average rate of change for $\sqrt{x}$ from 1 to 4.

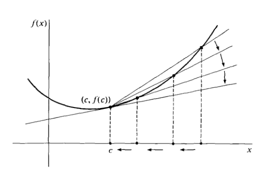
- Secant Line

  Using two points on of the function f you can form a secant line. The slope of the secant line is

$$m_{sec} = \frac{f(b) - f(a)}{b - a} = \frac{f(a+h) - f(a)}{h}$$



Using the Secant Line to Approximate the tangent line leads to Calculus



▼ Example 1: $f(x) = 2x^2 - 3x + 4$

a) Express the slope of the secant line in terms of x and h.

b) Find the slope of the secant line for $h = 0.5, 0.1, 0.001$ at $x = 1$

c) What value does the slope of the secant line approach as h approaches 0?

d) Find the equation of the secant line at $x = 1$ and $h = 0.01$,

e) Graph f and the secant line in the same viewing window.

▼ Example 2: $f(x) = \frac{9}{x}$

a) Express the slope of the secant line in terms of x and h.

b) Find the slope of the secant line for $h = \frac{1}{2}, \frac{1}{10}, \frac{1}{100}$ at $x = 1$

c) What value does the slope of the secant line approach as h approaches 0?

d) Find the equation of the secant line at $x = 1$ and $h = \frac{1}{100}$,

e) Graph f and the secant line in the same viewing window.