3.1 Functions

- Finding the Domain of a Function Given it's Equation
  - Polynomial Functions
    - Definition of a Polynomial Function

      A polynomial function is of the form

      $$f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$$

      where n is a non-negative integer, such as $\{0, 1, 2, ...\}$ and the coefficients $\{a_n, a_{n-1}, ..., a_2, a_1, a_0\}$ are real numbers

    - Practical Identification of Polynomial Functions

      Look for the variable to be in the numerator. (If there is no fraction at all, the terms are in the numerator.) The variable should not be inside a radical or absolute value. The powers or exponents should be whole numbers. Whole numbers come from this list. $\{0, 1, 2, ...\}$

    - Finding Domain of Polynomial Functions

      There are no restrictions to the domain of a polynomial function. Meaning that each x in the real numbers will result in a y in the real numbers. For that reason the domain of all polynomials is all real numbers. In interval notation, that is $(-\infty, \infty)$.

    - Examples of Polynomial Functions

      $f(x) = 2x - 7$

      $g(x) = 2x^2 - 9x + 5$

      $h(x) = 7x^3 + 5x^7$

      $s(t) = 5$

- Rational Functions
  - Definition of a Rational Function

    A rational function is of the form

$$f(x) = \frac{p(x)}{q(x)}$$

    where $p(x)$ and $q(x)$ are polynomial functions and $q(x)$ is not equal to zero.

  - Practical Identification of Rational Functions

    The numerator and denominator are polynomials. Most functions with variables in the denominator are considered rational functions but there are exceptions.Since rational functions are in the form of a fraction, there is a restriction that the denominator is not zero. You can find the restricted values be creating an equation. The equation is the denominator equal to zero.

  - Finding Domain of Rational Functions

    Since rational functions are in the form of a fraction, there is a restriction that the denominator is not zero. You can find the restricted values be creating an equation. The equation is the denominator equal to zero.

    $q(x) = 0$

    The solutions to this equation are restricted values and must be excluded from the domain.

  - Examples of Rational Functions

    $f(x) = \frac{2}{x-2}$

    $g(x) = \frac{x-5}{x^2-9}$

    $h(x) = \frac{2x}{6x^2-23x+20}$

    $s(t) = \frac{t^2+16}{t^3+5t^2+6t}$

- Radical\Root Functions (Even Index)
  - Definition of a Radical\Root Function (Even Index)

    A root function is of the form

    $$f(x) = \sqrt[n]{g(x)}$$

    where n is an even positive integer greater than or equal to 2.
  - Practical Identification of Radical\Root Functions (Even Index)

    The variable is inside or under the radical. The index of the radical is an even number $\{2, 4, 6, ...\}$. The square root is an even index although the index is not written.

    It is also possible to write radicals as fractional exponents.
  - Finding Domain of Radical\Root Functions (Even Index)

    Taking the even root of a negative number results in a complex or imaginary number. Since we are interested in real function values we would like the expression inside the radical to be non-negative (zero or positive). Create an inequality to solve. The inequality is the expression under the radical greater than or equal to zero.

    $g(x) \geq 0$

    The solution set of this inequality is the domain.
  - Examples of Radical\Root Functions (Even Index)

    $f(x) = \sqrt{x-5}$

    $g(x) = \sqrt[4]{6-3x}$

    $h(x) = \sqrt[6]{x^2-3x+5}$

    $s(t) = (2t-3)^{1/2}$

- Radical\Root Functions (Odd Index)
  - Definition of a Radical\Root Function (Odd Index)

    A root function is of the form

    $$f(x) = \sqrt[n]{g(x)}$$

    where n is an odd positive integer greater than or equal to 2.
  - Practical Identification of Radical\Root Functions (Odd Index)

    The variable is inside or under the radical. The index of the radical is an odd number. $\{3, 5, 7, ...\}$ The cube root is an odd index.

    It is also possible to write radicals as fractional exponents.
  - Finding Domain of Radical\Root Functions (Odd Index)

    There are no restrictions to the domain of root function with odd index. For that reason the domain of all root function with odd index is all real numbers. In interval notation, that is $(-\infty, \infty)$
  - Examples of Radical\Root Functions (Odd Index)

    $f(x) = \sqrt[3]{x-5}$

    $g(x) = \sqrt[5]{6-3x}$

    $h(x) = \sqrt[7]{x}$

    $s(t) = (2t-3)^{1/3}$

Find the Domain of Combined Functions

- The domain of $(f+g)(x)$ is the domain of f intersected with the domain of g. Symbolically, $D_{f+g} = D_f \cap D_g$
- The domain of $(f-g)(x)$ is the domain of f intersected with the domain of g. Symbolically, $D_{f-g} = D_f \cap D_g$
- The domain of $(fg)(x)$ is the domain of f intersected with the domain of g. Symbolically, $D_{fg} = D_f \cap D_g$
- The domain of $\left(\frac{f}{g}\right)(x)$ is the domain of f intersected with the domain of g and $g(x) \neq 0$. Symbolically, $D_{\frac{f}{g}} = D_f \cap D_g$ and $g(x) \neq 0$

- Given $f(x) = x^2 - 2x - 3$ and $g(x) = x^2 - 9$
  - Find the Domain of the functions f and g
  - Example 1: Find the Domain of $(f+g)(x) = 2x^2 - 2x - 12$
  - Example 2: Find the Domain of $(f-g)(x) = -2x + 6$
  - Example 3: Find the Domain of $(fg)(x) = x^4 - 2x^3 - 12x^2 + 18x + 27$
  - Example 4: Find the Domain of $\left(\frac{f}{g}\right)(x) = \frac{x^2-2x-3}{x^2-9} = \frac{x+1}{x+3}$
- Given $f(x) = \frac{1}{x-3}$ and $g(x) = \sqrt{x+4}$
  - Find the domain of the functions f and g
  - Example 5: Find the Domain of $(f+g)(x) = \frac{1}{x-3} + \sqrt{x+4}$
  - Example 6: Find the Domain of $(f-g)(x) = \frac{1}{x-3} - \sqrt{x+4}$
  - Example 7: Find the Domain of $(fg)(x) = \frac{\sqrt{x+4}}{x-3}$
  - Example 8: Find the Domain of $\left(\frac{f}{g}\right)(x) = \frac{1}{(x-3)\sqrt{x+4}}$

- Applications of Combined Functions
  - Revenue

$$R = xp$$

R-revenue
x-number of units
p-price

- Example: Build and evaluate the revenue function

The price p and the quantity x sold of a small flat-screen television set obeys the demand equation $p = -.16x + 240$

a) Express the revenue R as a function x. Use the formula $R = xp$.

b) How much should be charged for the television set if there are 60 television sets in stock?

c) How much should be charged for the television set if there are 750 television sets in stock?

d) What is the revenue when there are 750 sets in stock?

▼ Profit

$$P = R - C$$

P-profit
R-revenue
C-cost

▼ Example: Build and evaluate the revenue function

Rite-Cut riding lawnmowers obey the demand equation $p = -\frac{1}{20}x + 1010.$ The cost of producing x lawnmowers is given by the function $C(x) = 130x + 7000.$

a) Express the revenue R as a function x. Use the formula $R = xp$.

b) Express the profit P as a function x. Use the formula $P = R - C$.

c) What is the revenue if 8800 units are sold?

d) What is the profit if 8800 units are sold?

e) What price should be charged if 8800 units are sold?

▼ Rectangle

$$A = lw$$

$$P = 2l + 2w$$

A-area
l-length
w-width

P-Perimeter
l-length
w-width

▼ Example: Build and evaluate the area function

A farmer has 1,556 feet of fencing available to enclose a rectangular area bordering a river. No fencing is required along the river. Let x represent the length of the side of the rectangular enclosure that is perpendicular to the river.

a) Create an expression that represents the side parallel to the river.

b) What is the length of the side parallel to the river if the side perpendicular to the river is 389?

c) Create a function, A(x), that describes the total area of the rectangular function of x, where x is the length of the rectangular enclosure that is perpendicular to the river.

d) What is the area of the rectangle when the side perpendicular to the river is 389?

- The Difference Quotient

$$\frac{f(x+h)-f(x)}{h}$$

  - Examples: Calculating the Difference Quotient
    - Example 1: $f(x) = 2x - 3$
    - Example 2: $f(x) = 3x^2 - 5x + 7$
    - Example 3: $f(x) = \frac{1}{x}$
    - Example 4: $f(x) = \sqrt{x}$

- ▼ Summary of Strategies
    - ▼ Polynomial (use when you have polynomials like $f(x) = 2x - 3$ and $f(x) = 3x^2 - 5x + 7$)

      Simplify by removing parenthesis and combining like terms in the numerator. When you have simplified the numerator completely all of the terms remaining in the numerator will have an h. Cancel the common h from each term in the numerator.

    - ▼ Clearing Fractions (use when you have complex fractions like $f(x) = \frac{1}{x}$)

      Find the common denominator of the fractions and multiply the numerator and denominator by the common denominator. This will clear the fractions and allow you to use the dividing out method to cancel common factors and then finish with direct substitution.

    - ▼ Rationalize (use when you have radicals like $f(x) = \sqrt{x}$)

      Multiply by the conjugate of the radical expression on both the numerator and denominator. Once simplified you will be able to use the dividing out method to cancel common factors and then finish with direct substitution.