Finding Domain(Solutions)

Type of Function	Formal Definition	Practical Identification	Finding Domain
Polynomial	A polynomial function is of the form $f(x)=a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$ where n is a non-negative integer, such as {0, 1, 2, …} and the coefficients $a_n, a_{n-1}, ..., a_1, a_0$ are real numbers.	Look for the variable to be in the numerator. (If there is no fraction at all, the terms are in the numerator.) The variable should not be inside a radical or absolute value. The powers or exponents should be whole numbers. Whole numbers come from this list. {0, 1, 2, …}	There are no restrictions to the domain of a polynomial function. For that reason the domain of all polynomials is all real numbers. In interval notation, that is $(-\infty, \infty)$.
Rational	A rational function is of the form $f(x)=\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomial functions and $q(x)$ is not equal to zero.	The numerator and denominator are polynomials. Most functions with variables in the denominator are considered rational functions but there are exceptions.	Since rational functions are in the form of a fraction, there is a restriction that the denominator is not zero. You can find the restricted values be creating an equation. The equation is the denominator equal to zero. $q(x)=0$ The solutions to this equation are restricted values and must be excluded from the domain.
Radical/ Root with even index.	A root function is a function of the form $f(x)=\sqrt[n]{g(x)}$ where n is an even positive integer greater than or equal to 2.	The variable is inside or under the radical. The index of the radical is an even number {2, 4, 6, …}. The square root is an even index although the index is not written.	Taking the even root of a negative number results in a complex or imaginary number. Since we are interested in real function values we would like the expression inside the radical to be non-negative (zero or positive). Create an inequality to solve. The inequality is the expression under the radical greater than or equal to zero. $g(x)\geq 0$
Radical/ Root with odd index.	A root function is a function of the form $f(x)=\sqrt[n]{g(x)}$ where n is an odd positive integer greater than or equal to 2.	The variable is inside or under the radical. The index of the radical is an odd number. {3, 5, 7, …} The cube root is an odd index.	There are no restrictions to the domain of root function with odd index. For that reason the domain of all root function with odd index is all real numbers. In interval notation, that is $(-\infty, \infty)$.

Find the domain of the following functions.

1. $f(x)=\frac{2}{3}x-9$

Polynomial; $\mathbf{D:}(-\infty,\infty)$

2. $g(x)=2x^2-3x+4$

Polynomial; $\mathbf{D:}(-\infty,\infty)$

3. $h(x)=\sqrt{2}x^3-|-4|x^2-7x+2$

Polynomial; $\mathbf{D:}(-\infty,\infty)$

4. $R(x)=\frac{2x+3}{x-5}$

Rational; $\mathbf{D:}(-\infty,5)\cup(5,\infty)$

$$x-5\neq 0$$
$$x\neq 5$$

5. $P(x)=\frac{1}{x^2-1}$

Rational; $\mathbf{D:}(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

$$x^2-1\neq 0$$
$$(x+1)(x-1)\neq 0$$
$$x+1\neq 0 \text{ and } x-1\neq 0$$
$$x\neq -1 \text{ and } x\neq 1$$

6. $Q(x)=\frac{x}{x^3-2x^2-15x}$

Rational; $\mathbf{D:}(-\infty,-3)\cup(-3,0)\cup(0,5)\cup(5,\infty)$

$$x^3-2x^2-15x\neq 0$$
$$x(x-5)(x+3)\neq 0$$
$$x\neq 0 \text{ and } x-5\neq 0 \text{ and } x+3\neq 0$$
$$x\neq 0 \text{ and } x\neq 5 \text{ and } x\neq -3$$

7. $f(x)=\sqrt{x-9}$

Even Root; $\mathbf{D:}[9,\infty)$

$$x-9\geq 0$$
$$x\geq 9$$

8. $g(x)=\sqrt[4]{5-x}$

Even Root; $\mathbf{D:}(-\infty,5]$

$$5-x\geq 0$$
$$x\leq 5$$

9. $h(x)=\sqrt{x^2+x-6}$

Even Root; $\mathbf{D:}(-\infty,-3]\cup[2,\infty)$

$$x^2+x-6\geq 0$$

10. $R(x)=\sqrt[3]{2x+5}$

Odd Root; $\mathbf{D:}(-\infty,\infty)$

11. $P(x)=\sqrt[5]{x}$

Odd Root; $\mathbf{D:}(-\infty,\infty)$

12. $Q(x)=\sqrt[7]{x^2-9}$

Odd Root; $\mathbf{D:}(-\infty,\infty)$