Section 5.2 Guided Notebook

Section 5.2 Logarithmic Functions

- ☐ Work through Section 5.2 TTK #3
- ☐ Work through Section 5.2 TTK #4
- ☐ Work through Section 5.2 TTK #6
- ☐ Work through Section 5.2 TTK #9
- ☐ Work through Objective 1
- ☐ Work through Objective 2
- ☐ Work through Objective 3
- ☐ Work through Objective 4
- ☐ Work through Objective 5
- ☐ Work through Objective 6
- ☐ Work through Objective 7

Section 5.2 Logarithmic Functions

5.2 Things To Know

3. Determining Whether a Function is One-to-One Using the Horizontal Line Test
   Do you remember the **definition of a one-to-one function**? Go back to Section 3.6 to see this definition or work through the video. You should also work through the animation to recall how to use the horizontal line test. Pay close attention to the graph in part (b) of this animation. Does the graph in part (b) of the animation represent a one-to-one function?

4. Sketching the Graph of an Inverse Function
   Work through the animation and sketch the graph of the function $f(x) = x^2 + 1,\ x \le 0$ and sketch the graph of its inverse function. Also, state the domain and range of f and f^{-1}.

6. Finding the Equation of an Inverse Function
   Do you remember how to find the inverse of a one-to-one function? Work through the animation to find the inverse of $f(x) = \dfrac{2x}{1-5x}$ and state the domain and range of f and f^{-1}.

9. Solving Exponential Equations by Relating the Bases
   Work through the animation and solve the following exponential equations/

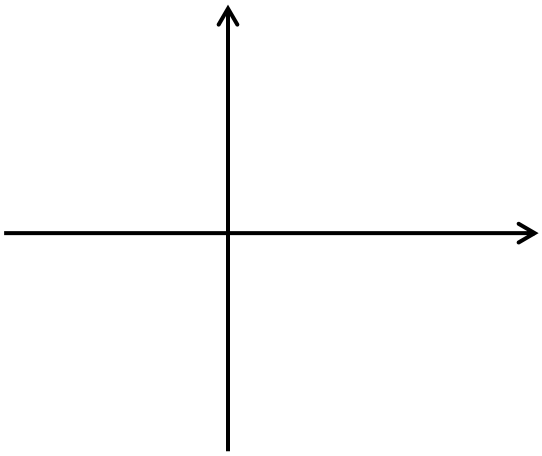
   a. $8 = \dfrac{1}{16^x}$

   b. $\dfrac{1}{27^x} = \left(\sqrt[4]{3}\right)^{x-2}$

Section 5.2 Objective 1 Understanding the Definition of a Logarithmic Function

Work through the video that accompanies Objective 1 and take notes here:

Sketch the graph of $f(x)=b^x, b>1$ as seen in the video and plot several points that lie on the graph, then sketch the graph of the inverse function.



We want to find the equation of the inverse of $f(x)=b^x$. Follow the 4-step process for finding inverse functions as stated in this video and see if we can find the equation of this inverse function.

Step 1:

Step 2:

Step 3:

Before we can complete Step 3, we must define the logarithmic function. Write the definition of a logarithmic function here:

Definition of the Logarithmic Function

Now Finish Step 3:

Step 4:

Section 5.2

Work through the video that accompanies Example 1:

Write each exponential equation as an equation involving a logarithm.

a. $2^3 = 8$

b. $5^{-2} = \frac{1}{25}$

c. $1.1^M = z$

Work through the video that accompanies Example 2:

Write each exponential equation as an equation involving an exponent.

a. $\log_3 81 = 4$

b. $\log_4 16 = y$

c. $\log_{3/5} x = 2$

Section 5.2 Objective 2 Evaluating Logarithmic Expressions

Read through Objective 2 in your eText and take notes here.

Explain how to evaluate the expression $\log_4 64$.

Write down "The Method of Relating the Bases".

Work through the interactive video that accompanies Example 3 and take notes here:

Evaluate each logarithm:

a. $\log_5 25$

b. $\log_3 \frac{1}{27}$

c. $\log_{\sqrt{2}} \frac{1}{4}$

Section 5.2 Objective 3 Understanding the Properties of Logarithms

Write down the two General Properties of Logarithms.

General Properties of Logarithms

For $b > 0$ and $b \neq 1$,

1.

2.

In Section 3.6, we studied one-to-one functions. Given a function f and the inverse function f^{-1}, we learned two composition cancellation equations. Write down the two composition cancellation equations here:

Composition Cancellation Equations

1.

2.

If $f(x) = b^x$ is an exponential function, the inverse function is $f^{-1}(x) = \log_b x$. Using this information and applying the two composition cancellation equations seen above, write down the two cancellation properties of exponentials and logarithms.

Cancellation Properties of Exponentials and Logarithms

For $b > 0$ and $b \neq 1$,

1.

2.

Work through Example 4:

Use the properties of logarithms that you have written above to evaluate each expression.

a. $\log_3 3^4$

b. $\log_{12} 12$

c. $7^{\log_7 13}$

d. $\log_8 1$

Section 5.2 Objective 4 Using the Common and Natural Logarithms

Read Objective 4 in your eText and take notes here. Then write down the definition of the **common logarithmic function** and the **natural logarithmic function.**

Common Logarithmic Function

Natural Logarithmic Function

Work through the video that accompanies Example 5:

Write each exponential equation as an equation involving a common logarithm or natural logarithm.

a. $e^0 = 1$

b. $10^{-2} = \frac{1}{100}$

c. $e^K = w$

Work through the video that accompanies Example 6:

Write each logarithmic equation as an equation involving an exponent.

a. $\log 10 = 1$

b. $\ln 20 = Z$

c. $\log(x-1) = T$

Work through the video that accompanies Example 7:

Evaluate each expression without the use of a calculator.
a. $\log 100$

b. $\ln \sqrt{e}$

c. $e^{\ln 51}$

d. $\log 1$

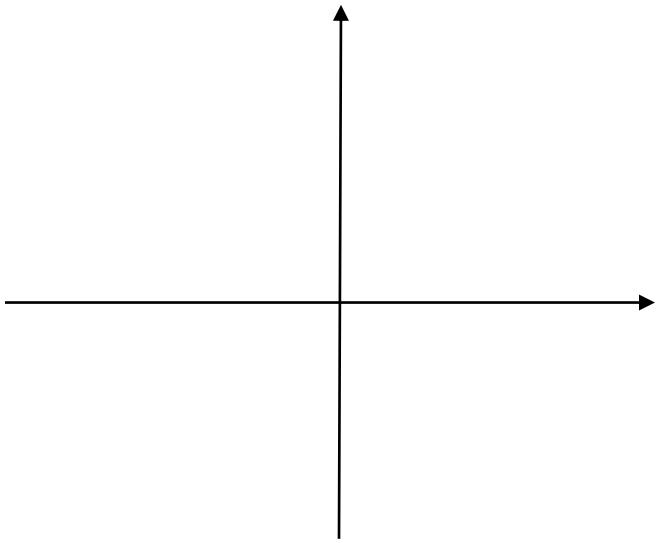
Section 5.2 Objective 5 Understanding the Characteristics of Logarithmic Functions
Write down the three steps to sketching the graph of a logarithmic function of the form $f(x) = \log_b x,\ b > 0, b \neq 1.$

Step 1:

Step 2:

Step 3:

Follow the 3 steps on the previous page to sketch the graph of $f(x) = \log_3 x$. Work through the video that accompanies Example 8 to see how to sketch this function.



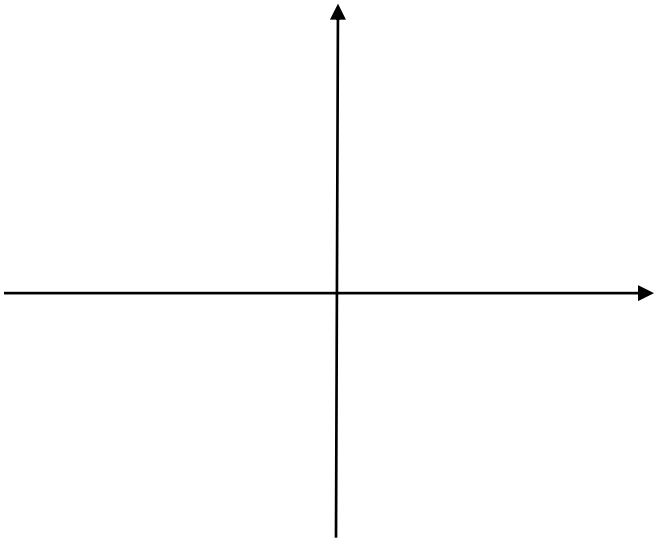
Write down the **Characteristics of Logarithmic Functions** as seen in your eText. Make sure to sketch the two graphs as seen on page 5.2-18 and plot the points as shown on the graphs.

Section 5.2 Objective 6 Sketching the Graphs of Logarithmic Functions Using Transformations

Do you remember the transformation techniques (how to shift the graphs of functions) that were discussed in Section 3.4? You may want to review these techniques before working on the next set of exercises.

Work through the video that accompanies Example 9:

Sketch the graph of $f(x) = -\ln(x+2) - 1$. (Make sure that your final graph contains the three points that are seen on the final graph as shown in the video.)



Take some time to experiment with the **Guided Visualization** titled "Sketching Logarithmic Functions" seen on page 5.2-21. Sketch two logarithmic functions below. One function should a natural logarithm and the other function should be a base other than *e*. Clearly label your functions and give the equations of the horizontal asymptotes.

Section 5.2 Objective 7 Finding the Domain of Logarithmic Functions

Look at your graph of $f(x) = -\ln(x+2) - 1$ from Example 9. What is the domain of this function? Do you see that the domain is $(-2, \infty)$? To find the domain of a logarithmic function of the form $f(x) = \log_b[g(x)]$, we must find the solution to the inequality $g(x) > 0$.

Example: Find the domain of $f(x) = \log_7[2x-5]$. To find the domain, we must solve the inequality $2x - 5 > 0$. Solving this inequality, we get $x > \frac{5}{2}$. So the domain of $f(x) = \log_7[2x-5]$ is $\left(\frac{5}{2}, \infty\right)$.

Now work through the interactive video that accompanies Example 10:

Find the domain of $f(x) = \log_5\left(\frac{2x-1}{x+3}\right)$.