Section 4.6 Guided Notebook

Section 4.6 Rational Functions and Their Graphs

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Section 4.6 Rational Functions and Their Graphs

Work through the interactive video for You Try It TTK #1 and take notes on how to find the domain of the function in part c. of the video:

c. Find the domain of $f(x) = \dfrac{x}{x^2 - x - 6}$.

Work through the video for You Try It TTK #2 **Determining Whether a Function is Even, Odd, or Neither** and take notes here.

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Work through the animation for You Try It TTK #3 **Using Combinations of Transformations to Graph Functions** and take notes here.

Introduction to Section 4.6

Write down the definition of a **Rational Function**.

Section 4.6 Objective 1: Finding the Domain and Intercepts of Rational Functions

What is the domain of a rational function?

How do you find the y-intercept of a rational function if there is one?

How do you find the x-intercepts of a rational function?

Work Example 1 showing all work below. To see the steps of the solution watch the video with the example.

Let $f(x)=\dfrac{x-4}{x^2+x-6}$.

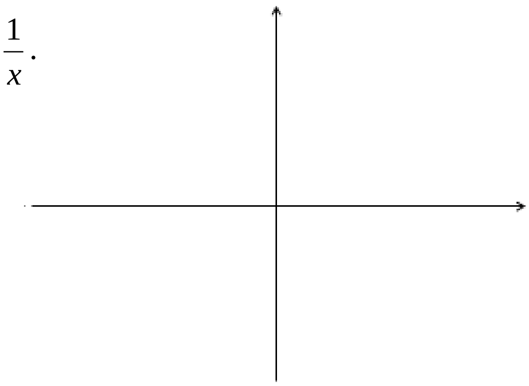
a. Determine the domain of *f*.

b. Determine the y-intercept (if any).

c. Determine any x-intercepts.

Section 4.6 Objective 2: Identifying Vertical Asymptotes

Sketch the reciprocal function, $f(x)=\dfrac{1}{x}$.



Look at the graph of $f(x)=\dfrac{1}{x}$ and fill in the blanks below:

The values of f approach ________ as x approaches _________ from the right.

The values of f approach ________ as x approaches _________ from the left.

Write down the definition of a **vertical asymptote** and sketch the four graphs as seen in the definition.

What is the caution statement say about locating vertical asymptotes?

Watch the video with Example 2 and answer the following questions.

Find the vertical asymptotes (if any) of the function $f(x)=\dfrac{x-3}{x^2+x-6}$ and then sketch the graph near the vertical asymptotes.

1. What is the first step?

2. What are the vertical asymptotes?

3. When graphing vertical asymptotes the lines must be ___________?

4. How do you sketch the graph near the asymptotes?

5. Show the work for each of the test values below. Include a sketch.

6. What is the side note about choosing the values and the *x*-intercept?

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Work through the video with Example 3 and take notes below.
Find the vertical asymptotes (if any) of the following function and then sketch the graph near the vertical asymptotes.

$$f(x)=\frac{x+3}{x^2+x-6}$$

1. What is the first step?

2. What happens to the common factor?

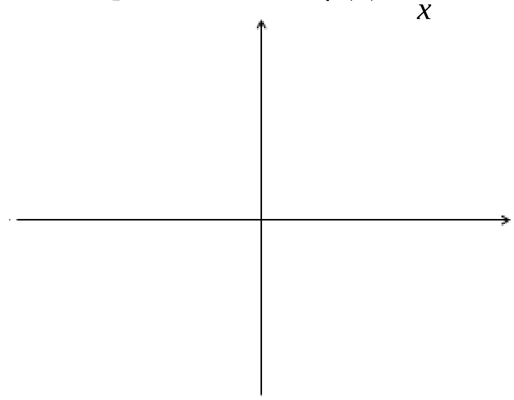
3. What is the vertical asymptote?

4. Show all steps to sketch the graph near the asymptote.

Section 4.6 Objective 3: Identifying Horizontal Asymptotes

Watch the video with Objective 3 and answer the following questions.

1. Sketch the graph of the reciprocal function $f(x) = \frac{1}{x}$.



2. What do you notice about the graph and the *y*-axis?

3. What do you notice about the graph and the *x*-axis?

4. What is the *x*-axis?

5. Write the definition of a **Horizontal Asymptote**.

6. Draw the illustration for the first 2 properties of **Horizontal Asymptotes of Rational Functions**.

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Write down the three Properties of **Horizontal Asymptotes of Rational Functions**.
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Finding Horizontal Asymptotes of a Rational Function

Let $f(x) = \dfrac{g(x)}{h(x)} = \dfrac{a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + b_{m-2}x^{m-2} + \cdots + b_1 x + b_0}$, $a_n \neq 0$, $b_m \neq 0$, where f is written in lowest terms, n is the degree of g, and m is the degree of h.

- If $m > n$, then $y = 0$ is the horizontal asymptote.
- If $m = n$, then the horizontal asymptote is $y = \dfrac{a_n}{b_m}$, the ratio of the leading coefficients.
- If $m < n$, then there are no horizontal asymptotes.

Work through Example 4 and show all work below. For detailed step by step solutions watch the video.

Find the horizontal asymptote of the graph of each rational function or state that one does not exist.

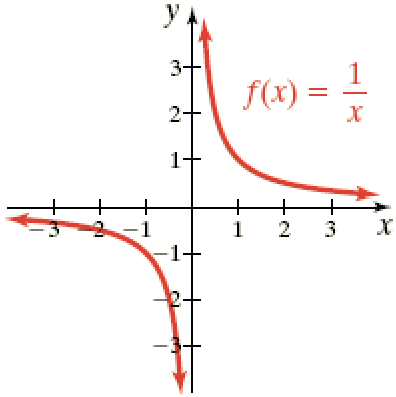
a. $f(x) = \dfrac{x}{x^2 - 4}$

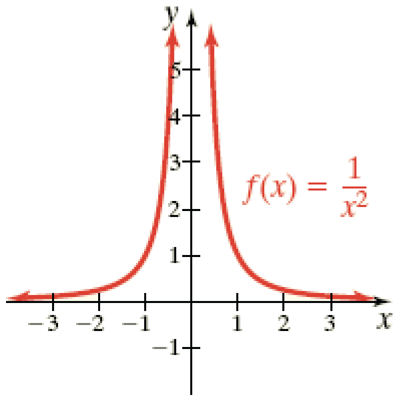
b. $f(x) = \dfrac{4x^2 - x + 1}{1 - 2x^2}$

c. $f(x) = \dfrac{2x^3 + 3x^2 - 2x - 2}{x - 1}$

Section 4.6 Objective 4: Using Transformations to Sketch the graphs of Rational Functions.

Write down the properties of $f(x)=\dfrac{1}{x}$ and $f(x)=\dfrac{1}{x^2}$.





Properties of the graph of $f(x)=\dfrac{1}{x}$

Properties of the graph of $f(x)=\dfrac{1}{x^2}$

Work through Example 5 showing all work below. The video will show the step by step solution.

Use transformations to sketch the graph of $f(x)=\dfrac{-2}{(x+3)^2}+1$.

Section 4.6 Objective 5: Sketching Rational Functions Having Removable Discontinuities

What are removable discontinuities? When do they occur?

Work through Example 6 showing all work below. For a detailed step by step solution watch the video.

Sketch the graph of the following function and find the coordinates of all removable discontinuities. $f(x)=\dfrac{x^2-1}{x+1}$.

Work through Example 7 showing all work below. For a detailed step by step solution watch the video with the example.

Sketch the graph of the following function and find the coordinates of all removable discontinuities. $f(x)=\dfrac{x+3}{x^2+x-6}$.

What is the caution statement about graphing calculators and removable discontinuities?

Section 4.6 Objective 6: Identifying Slant Asymptotes

When does a slant asymptote occur?

How do you find the slant asymptote?

Watch the video with Example 8 and take notes here.

Find the slant asymptote of $f(x)=\dfrac{2x^2+3x-2}{x-1}$.

Section 4.6 Objective 7: Sketching Rational Functions

What are the steps for Graphing Rational Functions of the Form $f(x)=\dfrac{g(x)}{h(x)}$?

Step 1.

Step 2.

Step 3.

Step 4.

Step 5.

Step 6.

Step 7.

Step 8.

Step 9.

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Watch the video with Example 9 answering the following questions.

Sketch the graph of $f(x)=\dfrac{x^3+2x^2-9x-18}{x^3+6x^2+5x-12}$.

1. Write down the steps to find the domain.

2. After finding the domain, what is the next step? Show all work for this step below.

3. What is the simplified function?

4. What happens at x = -3?

5. Is the function even or odd? What does that indicate about the graph?

6. Why is it a good idea to test for symmetry?

7. Show the steps to find the intercepts below.

8. Which denominator is used to find the vertical asymptotes? What are they?

9. Verify the behavior near the vertical asymptotes below.

10. Will there be a horizontal or slant asymptote? Why?

11. What is the horizontal or slant asymptote? How was it found?

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12. How do you determine which values of x to evaluate?

13. Show the steps to evaluate the function at these values.

14. Draw the final sketch of the graph below.