## Review Exam 2 STA2023

## Chapter 4 - Chapter 6

## Chapter 4 Discrete Probability Distributions

1. Write the sample space of rolling two dice.

Sample Space: $\{\quad(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$, $(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$, $(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$, $(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$, $(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$, $(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$
2. What are the possible values of the random variable $X$ if $X$ is defined as the sum of the two dice?

Sum of 2: $\quad(1,1)$
Sum of 3: $\quad(1,2),(2,1)$
Sum of 4: $\quad(1,3),(2,2),(3,1)$
Sum of 5: $\quad(1,4),(2,3),(3,2),(4,1)$
Sum of 6: $\quad(1,5),(2,4),(3,3),(4,2),(5,1)$
Sum of 7: $\quad(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)$
Sum of 8: $\quad(2,6),(3,5),(4,4),(5,3),(6,2)$
Sum of 9: $\quad(3,6),(4,5),(5,4),(6,3)$
Sum of 10: $\quad(4,6),(5,5),(6,4)$
Sum of 11: $(5,6),(6,5)$
Sum of 12: $(6,6)$
All possible sums for two standard dice.
$\mathrm{X}=2,3,4,5,6,7,8,9,10,11,12$
3. Build a probability distribution for the random variable $X$ defined in number 2.

| X | $\mathrm{P}(\mathrm{X})$ |
| :---: | :---: |
| 2 | $1 / 36$ |
| 3 | $2 / 36$ |
| 4 | $3 / 36$ |
| 5 | $4 / 36$ |
| 6 | $5 / 36$ |
| 7 | $6 / 36$ |
| 8 | $5 / 36$ |
| 9 | $4 / 36$ |
| 10 | $3 / 36$ |
| 11 | $2 / 36$ |
| 12 | $1 / 36$ |

4. Draw a probability histogram.

5. A ten sided die has the following faces: $\{1,2,2,3,3,3,4,6,7,7\}$. What are the possible values of the random variable $X$ if $X$ is defined as the number showing on the face of the die?
$\mathrm{X}=1,2,3,4,6,7$
6. Build a probability distribution for the random variable $X$ defined in number 16 .

| X | $\mathrm{P}(\mathrm{X})$ |
| :---: | :---: |
| 1 | $1 / 10$ |
| 2 | $2 / 10$ |
| 3 | $3 / 10$ |
| 4 | $1 / 10$ |
| 6 | $1 / 10$ |
| 7 | $2 / 10$ |

7. Draw a probability histogram.

8. Are the following random variables discrete or continuous?
a) The volume of coke in a one liter coke bottle continuous
b) The number of people who visit Wal-Mart on Sunday 11/24/06. discrete
c) The time it takes to travel from home to school. continuous
d) The number of customers at Wendy's at lunch time. discrete
9. Decide whether the distribution is a probability distribution. If it is not a probability distribution, identify the property or properties that are not satisfied.
a) This is a probability distribution

| x | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{x})$ | 0.05 | 0.25 | 0.35 | 0.25 | 0.1 |

b) This is not a probability distribution since the sum of the probabilities is more than 1.

| x | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{x})$ | 0.01 | 0.44 | 0.56 | 0.21 |

c) This is a probability distribution.

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{x})$ | 0.3 | 0.25 | 0.25 | 0.1 | 0.05 | 0.03 | 0.02 |

10. Determine the probability distributions missing value.

| $X$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $P(X)$ | 0.2 | 0.4 | 0.05 | $?$ | 0.25 |

$\underline{P}(\mathrm{X}=3)=0.1$ It must be this value so that the sum of the probabilities is 1.
11. Calculate the mean, expected value, variance and standard deviation of the random variable X.

| X | $\mathrm{P}(\mathrm{X})$ | Mean $\mu=3.28$ | Expected Value $E(x)=3.28$ |
| :--- | ---: | :--- | :--- |
| 1 | 0.15 |  |  |
| 2 | 0.23 | standard deviation $\sigma=1.5$ |  |
| 3 | 0.16 |  |  |
| 4 | 0.11 | variance $\sigma^{2}=2.25$ |  |
| $\mathbf{5}$ | $\mathbf{0 . 3 5}$ |  |  |

12. A charity organization is selling $\$ 5$ raffle tickets as a part of a fund-raising program. The first prize is a boat valued at $\$ 5150$, and the second prize is a camping tent valued at $\$ 600$. The remaining 10 prizes are $\$ 50$ gift certificates. The number of tickets sold is 4000 . What is the expected value of your gain?

| X | $\mathrm{P}(\mathrm{X})$ |
| :---: | :---: |
| $-5($ losing ticket $)$ | $3988 / 4000$ |
| $5145\left(1^{\text {st }}\right.$ prize $)$ | $1 / 4000$ |
| $595\left(2^{\text {nd }}\right.$ prize $)$ | $1 / 4000$ |
| 45 (remaining prize $)$ | $10 / 4000$ |

Expected Gain $\quad \mathrm{E}(\mathrm{X})=-3.44$
13. There is a $5 \%$ chance of hitting the bull's-eye when playing darts. Nicholas throws 5 darts. $X$ is the number of bull's-eyes.
a) What is the probability that he hits at least one bull's-eye? 1 -binompdf( $5, .05,0)$
b)What is the probability that he hits exactly one bull's-eye? binompdf $(5, .05,1)$
c) What is the probability that he hits less than two bull's eyes? Binomcdf( $5, .05,1)$
d) Find the mean, variance and standard deviation of the random variable $X$.

$$
\mu=n p=5 * .05=.25 ; \quad \sigma^{2}=n p q=5 * .05 * .95=.2375 ; \quad \sigma=\sqrt{n p q}=\sqrt{5 * .05 * .95}=.4873
$$

14. Bags of plain M\&M's contain $24 \%$ blue candies. One candy is selected from each of 6 bags. The random variable represents the number of blue candies. The random variable X is defined a the number of blue M\&M's. The probability experiment is binomial.
a) What is $n ? \underline{6}$ What is $p ? .24$ What is $q ? .76$
b) What values can the random variable take? $\underline{X}=0,1,2,3,4,5,6$
c) Find the probability that exactly 5 of the candies selected are blue. Binompdf( $6, .24,5$ )
d) Find the probability that no more that 2 of the candies selected are blue. Binomcdf( $6, .24,2$ )
e) Find the probability that at least 4 of the candies selected are blue. 1 -Binomcdf( $6, .24,3$ )

## Chapter 5 Normal Probability Distribution

15. Find the indicated area under the standard normal curve. Draw a picture of the standard normal curve and shade the area you are finding.
a) To the left of $z=1.23$
normalcdf(-E99, 1.23,0,1)
b) To the right of $z=1.05$
normalcdf(1.05,E99,0,1)
c) In between $z=1.05$ and $z=1.23$
normalcdf( $1.05,1.23,0,1$ )
16. Find the indicated probability using the standard normal distribution. Draw a picture of the standar normal curve and shade the probability that you are finding.
a) $P(z>1.78)$
normalcdf( $1.78, \mathrm{E} 99,0,1$ )
b) $p(z<-0.26)$
c) $P(-2.57<z<0.92)$
normalcdf(-E99,-0.26,0,1)
normalcdf(-2.57,0.92,0,1)
17. Assume the random variable X is normally distributed with a mean of 24 and a standard deviation of 3 . Find the following probabilities. Draw a picture of the normal curve and shade the probability you are finding.
a) $P(X>24) \quad$ normalcdf(24,E99,24,3)
b) $p(X<22) \quad$ normalcdf(-E99,22,24,3)
c) $P(18<X<25.5) \quad$ normalcdf( $18,25.5,24,3)$
18. Suppose the scores for an entrance exam are normally distributed with a mean of 550 and a variance of 900 .
a. If you need a score of 575 to get into a certain college, what is the probability that you will get in. normalcdf(575,E99,550,30)
b. If instead you need a score of 540 , what is the probability that you will get in?
normalcdf(540,E99,550,30)
19. The top $15 \%$ of finishers (fastest time) in a triathlon receive awards. If the finisher's time is normally distributed with a mean of 150 minutes and a standard deviation of 15 minutes, find the slowest time you can have and still get an award.

InvNorm(.15, 150, 15)
20. Applicants that score in the top $10 \%$ of aptitude test are considered for a second round of interview. If the scores are normally distributed with an average score of 126 and a standard deviation of 35 . What is the minimum score a student would need to obtain to be considered for the second round interview?

InvNorm(.90, 126, 35)
21. As reported by the U.S National Center for Health Statistics, the mean serum high-densitylipoprotein (HDL) cholesterol of females 20-29 years old is 53 . If serum HDL cholesterol is normally distributed with a standard deviation of 13.4 , answer the following questions.
a) What is the probability that a randomly selected female 20-29 years old will have a serum cholesterol of above 60 ? normalcdf( $60, \mathrm{E} 99,53,13.4$ )
b) What is the probability a random sample of 15 female $20-29$ year olds will have a mean serum cholesterol above 60 ? normalcdf( $60, \mathrm{E} 99,53,3.46$ )
c) What is the probability a random sample of 20 female $20-29$ year olds will have a mean serum cholesterol above 60 ? normalcdf( $60, \mathrm{E} 99,53,3.0$ )
d) What effect does increasing the sample size have on the probability? Provide an explanation for this result. Increasing the sample size decreases the probability. As your sample increases the sample mean should become a better estimate for the population mean thus making the sample mean of 60 being less likley.

## Chapter 6 Confidence Intervals

22. Dr. Paul wants to estimate the mean serum HDL cholesterol of his 20-29 year old female patients. He randomly selects 15 of his patients and obtains the data shown below. Use the data to compute a point estimate for the population mean serum HDL cholesterol.

|  | 65 | 47 | 51 | 54 | 70 |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | 55 | 44 | 48 | 36 | 53 |
|  | 45 | 34 | 59 | 45 | 54 |

23. Calculate a point estimate for mean using the following data values.

| 34 | 22 | 16 | 64 | 34 |
| :--- | :--- | :--- | :--- | :--- |
| 44 | 45 | 54 | 35 | 64 |
| 45 | 34 | 59 | 45 | 54 |

$\bar{x}=43.3$
24. A store manager reports a confidence interval of $(44.07,80.97)$ when estimating the mean price (in dollars) of a sample of text books. Find the sample mean and margin of error.

$$
\bar{x}=62.52 \quad E=18.45
$$

25. From a random sample of 50 Harry Potter fans, it was reported that it took an average of 30 hours to read The Order of the Phoenix with a population standard deviation of 8 hours. Construct a $95 \%$ confidence interval for the average number of hours it takes to read the book.

## Use zinterval

26. You randomly select 15 English teachers and record the time it takes to read and correct a 1 page paper. The sample mean time is 10 minutes with a population a standard deviation of 2 minutes. The time it takes to read and correct a 1 page paper is normally distributed. Construct a $99 \%$ confidence interval for the mean number of minutes it takes to read and correct a 1 page paper. Assume the distribution is normal.

## Use zinterval

27. Determine the minimum sample size if you want to be $95 \%$ confident that the mean number of days in a year it will rain in Jacksonville is within 1 day. It is known from past studies that population standard deviation is 5 days.
Use $n=\left(\frac{z_{c} \sigma}{E}\right)^{2}$ and

| Level of Confidence | $z_{c}$ |
| :---: | :---: |
| 0.80 | 1.28 |
| 0.90 | 1.645 |
| 0.95 | 1.96 |
| 0.99 | 2.575 |

28. An admissions director wants to estimate the mean age of all students enrolled at a college. The estimate must be within 1 year of the population mean. Assume the population of ages is normally distributed.
a) Determine the minimum sample size to construct a $90 \%$ confidence interval for the population mean. Assume the population standard deviation is 1.2 years.
Use $n=\left(\frac{z_{c} \sigma}{E}\right)^{2}$ and

| Level of Confidence | $z_{c}$ |
| :---: | :---: |
| 0.80 | 1.28 |
| 0.90 | 1.645 |
| 0.95 | 1.96 |
| 0.99 | 2.575 |

b) Repeat part a with a $99 \%$ confidence interval.

Use $n=\left(\frac{z_{c} \sigma}{E}\right)^{2}$ and

| Level of Confidence | $z_{c}$ |
| :---: | :---: |
| 0.80 | 1.28 |
| 0.90 | 1.645 |
| 0.95 | 1.96 |
| 0.99 | 2.575 |

29. You want to estimate the mean repair cost for dishwashers. In a random sample of 34 dishwashers you find that the mean repair cost is $\$ 60$ dollars with a standard deviation of $\$ 22.50$. Construct a $99 \%$ confidence interval for the mean.

## Use Tinterval

30. Construct a $95 \%$ confidence interval for the population mean. In a random sample of 10 microwave ovens, the mean repair cost was $\$ 75.00$ and a standard deviation of $\$ 12.50$. Assume the microwave repair costs are normally distributed.

## Use Tinterval

31. In a random sample of 500 Harry Potter fans, it was reported that 289 will be attending the midnight showing of The Order of the Phoenix. Find a point estimate for the proportion of Harry Potter fans attending the midnight showing of the movie and construct a $90 \%$ confidence interval for the proportion.

## Use 1propzint

32. You wish to estimate, with $99 \%$ confidence, the population proportion of U.S. adults who decided not to get a flu vaccine. Your estimate must be accurate within $2 \%$ of the population proportion.
a. No preliminary estimate is available. Find the minimum sample size needed.

Use $n=\hat{p} \hat{q}\left(\frac{z_{c}}{E}\right)^{2}$ with $\hat{p}=0.5$ and $\hat{q}=0.5$ and

| Level of Confidence | $z_{c}$ |
| :---: | :---: |
| 0.80 | 1.28 |
| 0.90 | 1.645 |
| 0.95 | 1.96 |
| 0.99 | 2.575 |

b. Find the minimum sample size needed, using a prior study that found that $38 \%$ decided not to get the flu vaccine.
Use $n=\hat{p} \hat{q}\left(\frac{z_{c}}{E}\right)^{2}$ and Use

| Level of Confidence | $z_{c}$ |
| :---: | :---: |
| 0.80 | 1.28 |
| 0.90 | 1.645 |
| 0.95 | 1.96 |
| 0.99 | 2.575 |

