

MAC2311 Round Robin Ch3 Solutions

Station 1

$$f(x) = x^5 + \sin x + \cos x + \ln x + e^x$$

$$f'(x) = 5x^4 + \cos x - \sin x + \frac{1}{x} + e^x$$

Station 2

$$y = 5x^3 + \sqrt[3]{x} + \frac{1}{x^2} + \sqrt{x} + 8$$

$$y = 5x^3 + x^{1/3} + x^{-2} + x^{1/2} + 8$$

$$y' = 15x^2 + \frac{1}{3}x^{-2/3} - 2x^{-3} + \frac{1}{2}x^{-1/2} + 0$$

$$y' = 15x^2 + \frac{1}{3\sqrt[3]{x^2}} - \frac{2}{x^3} + \frac{1}{2\sqrt{x}}$$

Station 3

$$y = \sin(2x+1) + \cos(3x^2-5) + e^{7x+1} + \ln(5x-1) + (4x-3)^6$$

$$y' = \cos(2x+1) \cdot 2 - \sin(3x^2-5) \cdot 6x + e^{7x+1} \cdot 7 + \frac{5}{5x-1} + 6(4x-3)^5 \cdot 4$$

$$y' = 2\cos(2x+1) - 6x\sin(3x^2-5) + 7e^{7x+1} + \frac{5}{5x-1} + 24(4x-3)^5$$

Station 4

$$f(x) = x \sqrt{2x-3}$$

$$f(x) = x (2x-3)^{\frac{1}{2}}$$

$$f'(x) = x \cdot \frac{d}{dx} \left[(2x-3)^{\frac{1}{2}} \right] + (2x-3)^{\frac{1}{2}} \cdot \frac{d}{dx} [x]$$

$$= x \cdot \frac{1}{2} (2x-3)^{-\frac{1}{2}} \cdot 2 + (2x-3)^{\frac{1}{2}} \cdot 1$$

$$= x (2x-3)^{-\frac{1}{2}} + (2x-3)^{\frac{1}{2}}$$

$$= (2x-3)^{-\frac{1}{2}} \left(x + (2x-3)^{\frac{1}{2}-\left(-\frac{1}{2}\right)} \right)$$

$$= (2x-3)^{-\frac{1}{2}} \left(x + (2x-3)^{\frac{1}{2}+\frac{1}{2}} \right)$$

$$= (2x-3)^{-\frac{1}{2}} (x + (2x-3)^1)$$

$$= (2x-3)^{-\frac{1}{2}} (x + 2x-3)$$

$$= (2x-3)^{-\frac{1}{2}} (3x-3)$$

$$= \frac{3x-3}{\sqrt{2x-3}}$$

Station 5

$$\frac{d}{dx} \left[\frac{1+x^2}{1-x^2} \right]$$

$$\begin{aligned}\frac{d}{dx} \left[\frac{1+x^2}{1-x^2} \right] &= \frac{(1-x^2) \frac{d}{dx}(1+x^2) - (1+x^2) \frac{d}{dx}(1-x^2)}{(1-x^2)^2} \\&= \frac{(1-x^2)(2x) - (1+x^2)(-2x)}{(1-x^2)^2} \\&= \frac{2x - 2x^3 + 2x + 2x^3}{(1-x^2)^2} \\&= \frac{4x}{(1-x^2)^2}\end{aligned}$$

Station 6

$$y = x^2 \sin\left(\frac{1}{x}\right) = x^2 \sin(x^{-1})$$

$$\begin{aligned}y' &= x^2 \frac{d}{dx}(\sin(x^{-1})) + \sin(x^{-1}) \frac{d}{dx}(x^2) \\&= x^2 \cos(x^{-1}) \cdot \frac{d}{dx}(x^{-1}) + \sin(x^{-1})(2x) \\&= x^2 \cos(x^{-1}) \cdot -1x^{-2} + 2x \sin(x^{-1}) \\&= -\frac{x^2 \cos\left(\frac{1}{x}\right)}{x^2} + 2x \sin\left(\frac{1}{x}\right) \\&= -\cos\left(\frac{1}{x}\right) + 2x \sin\left(\frac{1}{x}\right)\end{aligned}$$

Station 7

Find the equation of the tangent line for
 $f(x) = 3x^2 - 2x + 7$ at $x = 1$.

$$f'(x) = 6x - 2$$

$$m = f'(1) = 6(1) - 2 = 6 - 2 = 4$$

$$f(1) = 3(1)^2 - 2(1) + 7 = 3(1) - 2 + 7 = 3 - 2 + 7 = 8$$

$$y - y_1 = m(x - x_1)$$

$$y - 8 = 4(x - 1)$$

Station 8

$$S(x) = 20 + 64x - 16x^2$$

Average Velocity from $x=0$ to $x=2$

$$S(0) = 20 + 64(0) - 16(0)^2 = 20$$

$$S(2) = 20 + 64(2) - 16(2)^2 = 20 + 128 - 64 = 84$$

$$\text{Average Velocity} = \frac{S(2) - S(0)}{2 - 0} = \frac{84 - 20}{2} = \frac{64}{2} = 32 \text{ ft/sec}$$

Velocity at $x=2$

$$V(x) = S'(x) = 64 - 32x$$

$$V(2) = S'(2) = 64 - 32(2) = 64 - 64 = 0 \text{ ft/sec}$$

Acceleration at $x=2$

$$a(x) = v'(x) = s''(t) = -32$$

$$a(2) = v'(2) = s''(2) = -32 \text{ ft/sec}^2$$

Station 9

$$\frac{dr}{dt} = 0.3 \text{ in/sec}$$

$$S = 100\pi$$

$$\frac{dv}{dt} = ?$$

$$S = 100\pi$$

$$\frac{4\pi r^2}{4\pi} = \frac{100\pi}{4\pi}$$

$$r^2 = 25$$

$$r = 5$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt}$$

?

?

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$$\frac{dv}{dt} = \frac{4}{3}\pi \cdot 3(5)^2 \cdot .3$$

$$\frac{dv}{dt} = 30\pi \text{ in/sec}$$

Station 10

$$x^3 - xy + y^3 = 1$$

$$\frac{d}{dx} [x^3 - xy + y^3] = \frac{d}{dx} [1]$$

$$3x^2 - (xy' + y(1)) + 3y^2 y' = 0$$

$$\begin{matrix} 3x^2 - xy' - y & + 3y^2 y' \\ -3x^2 & + y & -3x^2 + y \end{matrix}$$

$$-xy' + 3y^2 y' = y - 3x^2$$

$$3y^2 y' - xy' = y - 3x^2$$

$$\frac{y' (3y^2 - x)}{(3y^2 - x)} = \frac{y - 3x^2}{3y^2 - x}$$

$$y' = \frac{y - 3x^2}{3y^2 - x}$$

Station 11

$$y = 2^x + 3^{4x} + \log_5 x + \log_6 (7x+1)$$

$$D_x[y] = \ln 2 \cdot 2^x + \ln 3 \cdot 3^{4x} \cdot 4 + \frac{1}{\ln 5 x} + \frac{7}{\ln 6 (7x+1)}$$

$$= (\ln 2) 2^x + 4(\ln 3) \cdot 3^{4x} + \frac{1}{(\ln 5)x} + \frac{7}{(\ln 6)(7x+1)}$$

Station 12

$$\begin{aligned} f(x) &= 3 \tan x + \sec(4x) + \csc(x^2) + \cot^2(5x) \\ &= 3 \tan x + \sec(4x) + \csc(x^2) + (\cot(5x))^2 \end{aligned}$$

$$\begin{aligned} D_x[f] &= 3 \sec^2 x + \sec(4x) \tan(4x) \cdot (4) - \csc(x^2) \cot(x^2) \cdot (2x) + 2(\cot(5x))' (-\csc^2(5x)) \cdot 5 \\ &= 3 \sec^2 x + 4 \sec(4x) \tan(4x) - 2x \csc(x^2) \cot(x^2) - 10 \cot(5x) \csc^2(5x) \end{aligned}$$

Station 13

$$\begin{aligned} f(x) &= \arcsin(x^2) + \arccos(3x) + 2 \arctan(x) + 4 \text{arcsec}(x) + \text{arccsc}(5x) + \text{arcot}(1/x) \\ &= \arcsin(x^2) + \arccos(3x) + 2 \arctan x + 4 \text{arcsec}(x) + \text{arccsc}(5x) + \text{arcot}(x^{1/2}) \end{aligned}$$

$$\begin{aligned} f'(x) &= \frac{2x}{\sqrt{1-(x^2)^2}} - \frac{3}{\sqrt{1-(3x)^2}} + 2 \cdot \frac{1}{1+x^2} + 4 \cdot \frac{1}{|x|\sqrt{x^2-1}} - \frac{5}{|5x|\sqrt{(5x)^2-1}} - \frac{\frac{1}{2}x^{-1/2}}{1+(x^{1/2})^2} \\ &= \frac{2x}{\sqrt{1-x^4}} - \frac{3}{\sqrt{1-9x^2}} + \frac{2}{1+x^2} + \frac{4}{|x|\sqrt{x^2-1}} - \frac{5}{|5x|\sqrt{25x^2-1}} - \frac{1}{2\sqrt{x}(1+x)} \end{aligned}$$

Station 14

$$y = 5x^6 + 7x^5 - 8x^4 + 9x^3 + 3x^2 - 2x + 7$$

$$\frac{dy}{dx} = 30x^5 + 35x^4 - 24x^3 + 27x^2 + 6x - 2$$

$$\frac{d^2y}{dx^2} = 150x^4 + 140x^3 - 72x^2 + 54x + 6$$

$$\frac{d^3y}{dx^3} = 600x^3 + 420x^2 - 144x + 54$$