

24g.  $\int_4^{64} \frac{dt}{bt^4 \sqrt{\log_4 t}}$

$$u = \log_4 t$$

$$\frac{du}{dt} = \frac{1}{t \ln 4}$$

$$t \ln 4 du = dt$$

$$t = 4$$

$$u = \log_4 4 = 1$$

$$t = 64$$

$$u = \log_4 64 = 3$$

$$\int_4^{64} \frac{dt}{bt^4 \sqrt{\log_4 t}} = \int_1^3 \frac{t \ln 4 du}{bt^4 \sqrt{u}}$$

$$= \int_1^3 \frac{\ln 4 du}{b^4 \sqrt{u}} = \frac{\ln 4}{b} \int_1^3 \frac{du}{4\sqrt{u}}$$

$$= \frac{\ln 4}{b} \int_1^3 u^{-1/4} du = \frac{\ln 4}{b} \left[ \frac{u^{3/4}}{3/4} \right]_1^3$$

$$= \frac{\ln 4}{b} \cdot \frac{4}{3} \left[ u^{3/4} \right]_1^3 = \frac{4 \ln 4}{18} \left[ u^{3/4} \right]_1^3$$

$$= \frac{2 \ln 4}{9} \left[ u^{3/4} \right]_1^3 = \frac{2 \ln 4}{9} \left( 3^{3/4} - 1^{3/4} \right)$$

$$= \frac{2 \ln 4}{9} \left( 4\sqrt[3]{3} - 4\sqrt[3]{1} \right)$$

$$= \frac{2 \ln 4}{9} \left( 4\sqrt[3]{27} - 4\sqrt[3]{1} \right)$$

$$= \frac{2 \ln 4}{9} \left( 4\sqrt[3]{27} - 4 \right)$$

$$= \frac{\ln 16}{9} \left( 4\sqrt[3]{27} - 4 \right)$$

power rule  
for logs

$$\log_b M^p = p \log_b M$$

$$24 \text{ h. } \int \frac{\csc^2 \theta}{8+3\cot \theta} d\theta$$

MAC2311  
Test 4 Review

$$u = 8+3\cot \theta$$

$$\frac{du}{d\theta} = -3\csc^2 \theta$$

$$\frac{du}{-3\csc^2 \theta} = \frac{-3\csc^2 \theta d\theta}{-3\csc^2 \theta}$$

$$\frac{du}{-3\csc^2 \theta} = d\theta$$

$$\int \frac{\csc^2 \theta}{8+3\cot \theta} d\theta$$

$$= \int \frac{\csc^2 \theta}{u} \frac{du}{-3\csc^2 \theta}$$

$$= -\frac{1}{3} \int \frac{du}{u} = -\frac{1}{3} \ln|u| + C$$

$$= -\frac{1}{3} \ln|8+3\cot \theta| + C$$

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$$24 \text{ i. } \int \frac{4e^{2x}}{e^{2x}-4} dx = \int \frac{4e^{2x}}{u} \cdot \frac{du}{2e^{2x}} = 2 \int \frac{du}{u}$$

$$u = e^{2x}-4$$

$$\frac{du}{dx} = 2e^{2x}$$

$$\frac{du}{2e^{2x}} = \frac{2e^{2x} dx}{2e^{2x}}$$

$$\frac{du}{2e^{2x}} = dx$$

$$= 2 \ln|u| + C$$

$$= 2 \ln|e^{2x}-4| + C$$

$$= \ln(e^{2x}-4)^2 + C$$