(1a) $\lim_{x\to 2}(3x^2+5) = 3(2)^2+5 = 3(4)+5 = 12+5 = 17$

(1b) $\lim_{x\to -1}\frac{x^2+3x+2}{x^2+1} = \frac{(-1)^2+3(-1)+2}{(-1)^2+1} = \frac{1-3+2}{1+1} = \frac{0}{2} = 0$

(1c) $\lim_{x\to 3}\sqrt{x^2-4} = \sqrt{3^2-4} = \sqrt{9-5} = \sqrt{5}$

(1d) $\lim_{x\to 2}\frac{x-2}{x^2-4} = \lim_{x\to 2}\frac{x-2}{(x+2)(x-2)} = \lim_{x\to 2}\frac{1}{x+2} = \frac{1}{2+2} = \frac{1}{4}$

(1e) $\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x} = \lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}\cdot\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \lim_{x\to 0}\frac{x+4-4}{x(\sqrt{x+4}+2)} = \lim_{x\to 0}\frac{x}{x(\sqrt{x+4}+2)}$

$= \lim_{x\to 0}\frac{1}{\sqrt{x+4}+2} = \frac{1}{\sqrt{0+4}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}$

(1f) $\lim_{x\to 1}\frac{x^2+x-2}{x^2-3x+2} = \lim_{x\to 1}\frac{(x+2)(x-1)}{(x-2)(x-1)} = \lim_{x\to 1}\frac{x+2}{x-2} = \frac{1+2}{1-2} = \frac{3}{-1} = -3$

(1g) $\lim_{x\to 0}\frac{\sin x}{x^2}$ use Squeeze theorem

$$-1 \le \sin x \le 1$$

$$-\frac{1}{x^2} \le \frac{\sin x}{x^2} \le \frac{1}{x^2}$$

$$\lim_{x\to 0}-\frac{1}{x^2} \le \lim_{x\to 0}\frac{\sin x}{x^2} \le \lim_{x\to 0}\frac{1}{x^2}$$

$$0 \le \lim_{x\to 0}\frac{\sin x}{x^2} \le 0$$

thus $\lim_{x\to 0}\frac{\sin x}{x^2} = 0$

(1h) $\lim_{x\to 5}\frac{x-5}{|x-5|}$ Look at the one sided limits since $|x-5| = \begin{cases} x-5 & x \ge 5 \\ -(x-5) & x < 5 \end{cases}$

$\lim_{x\to 5^+}\frac{x-5}{|x-5|} = \lim_{x\to 5^+}\frac{x-5}{x-5} = \lim_{x\to 5^+} 1 = 1$

$\lim_{x\to 5^-}\frac{x-5}{|x-5|} = \lim_{x\to 5^-}\frac{x-5}{-(x-5)} = \lim_{x\to 5^-} -1 = -1$ > Don't agree

thus $\lim_{x\to 5}\frac{x-5}{|x-5|}$ = DNE Does not exist

(i) $\lim_{x\to\infty} \frac{2x^2+x-2}{3x^2-3x+2} = \lim_{x\to\infty} \frac{\frac{2x^2+x-2}{x^2}}{\frac{3x^2-3x+2}{x^2}} = \lim_{x\to\infty} \frac{2+\frac{1}{x}-\frac{2}{x^2}}{3-\frac{3}{x}+\frac{2}{x^2}} = \frac{2+0-0}{3-0+0} = \frac{2}{3}$

or

$\lim_{x\to\infty} \frac{2x^2+x-2}{3x^2-3x+2} = \lim_{x\to\infty} \frac{4x+1}{6x-3} = \lim_{x\to\infty} \frac{4}{6} = \frac{4}{6} = \frac{2}{3}$

(j) $\lim_{x\to-\infty} \frac{\sqrt{4x+x^2}}{4x+1} = \lim_{x\to-\infty} \frac{\frac{\sqrt{4x+x^2}}{x}}{\frac{4x+1}{x}} = \lim_{x\to-\infty} \frac{\frac{\sqrt{4x+x^2}}{-\sqrt{x^2}}}{\frac{4x+1}{x}} = \lim_{x\to-\infty} \frac{-\sqrt{\frac{4x}{x^2}+\frac{x^2}{x^2}}}{\frac{4x}{x}+\frac{1}{x}}$

$= \lim_{x\to-\infty} \frac{-\sqrt{\frac{4}{x}+1}}{4+\frac{1}{x}} = \lim_{x\to-\infty} \frac{-\sqrt{\frac{4}{x}+1}}{4+\frac{1}{x}} = \frac{-\sqrt{0+1}}{4+0} = \frac{-\sqrt{1}}{4} = \frac{-1}{4}$

(k) $\lim_{x\to1} \frac{x^2+7x-8}{x-1} = \lim_{x\to1} \frac{(x-1)(x+8)}{x-1} = \lim_{x\to1} x+8 = 1+8 = 9$

(l) $\lim_{x\to0} \frac{\sin^2(3x)}{x^2} = \lim_{x\to0} \frac{\sin(3x)}{x} \cdot \frac{\sin(3x)}{x} = \lim_{x\to0} \frac{3\cdot3}{3\cdot3} \cdot \frac{\sin(3x)}{x} \cdot \frac{\sin(3x)}{x}$

$= \lim_{x\to0} 3\cdot3 \cdot \frac{\sin(3x)}{3x} \cdot \frac{\sin(3x)}{3x} = 9\cdot1\cdot1 = 9$

(m) $\lim_{x\to-1^-} \frac{x^3}{(x+1)^2}$ Direct sub $\frac{(-1)^3}{(-1+1)^2} = \frac{-1}{0^2} = \frac{-1}{0} \Rightarrow$ VA @ $x=-1$

limit will approach ∞ or $-\infty$

-1^- choose # less than -1

$\frac{(-2)^3}{(-2+1)^2} = \frac{-8}{(-1)^2} = -8 \Rightarrow -\infty$

$\lim_{x\to-1^-} \frac{x^3}{(x+1)^2} = -\infty$

(n) $\lim_{x\to3^+} \frac{x}{x-3}$ $\frac{3}{3-3} = \frac{3}{0} \Rightarrow$ VA @ $x=3$

3^+ choose # >3

$\frac{4}{4-3} = \frac{4}{1} = 4 \Rightarrow \infty$

$\lim_{x\to3^+} \frac{x}{x-3} = \infty$

(10) $\lim_{x\to 4} \frac{x^2+2x-7}{x-4}$ $\quad \frac{4^2+2(4)-7}{4-4} = \frac{16+8-7}{0} = \frac{17}{0} \Rightarrow$ VA @ $x=4$

$4^+ \quad \frac{5^2+2(5)-7}{5-4} = \frac{25+10-7}{1} = 35-7 = 28 \Rightarrow \infty$

$4^- \quad \frac{3^2+2(3)-7}{3-4} = \frac{9+6-7}{-1} = \frac{8}{-1} = -8 \Rightarrow -\infty$

$\lim_{x\to 4^+} \frac{x^2+2x-7}{x-4} = \infty \neq \lim_{x\to 4^-} \frac{x^2+2x-7}{x-4} = -\infty$ thus $\lim_{x\to 4} \frac{x^2+2x-7}{x-4} =$ DNE

(19) $\lim_{x\to 2^-} \frac{1}{x-2}$ $\quad \frac{1}{2-2} = \frac{1}{0} \Rightarrow$ VA @ $x=2$

$2^- \quad \frac{1}{1-2} = \frac{1}{-1} = -1 \Rightarrow -\infty$

$\lim_{x\to 2^-} \frac{1}{x-2} = -\infty$

(1r) $\lim_{x\to 2} \frac{1}{(x-2)^2}$ $\quad \frac{1}{(2-2)^2} = \frac{1}{0} \Rightarrow$ VA @ $x=2$

$2^+ \quad \frac{1}{(3-2)^2} = \frac{1}{1^2} = 1 \Rightarrow \infty$

$2^- \quad \frac{1}{(1-2)^2} = \frac{1}{(-1)^2} = 1 \Rightarrow \infty$

$\lim_{x\to 2^-} \frac{1}{(x-2)^2} = \infty = \lim_{x\to 2^+} \frac{1}{(x-2)^2}$ thus $\lim_{x\to 2} \frac{1}{(x-2)^2} = \infty$

(1s) $\lim_{x\to 0} \frac{\frac{1}{x+3} - \frac{1}{3}}{x} = \lim_{x\to 0} \frac{\frac{1}{x+3} - \frac{1}{3}}{x} \cdot \frac{3(x+3)}{3(x+3)} = \lim_{x\to 0} \frac{\frac{3(x+3)}{x+3} - \frac{3(x+3)}{3}}{3x(x+3)}$

$= \lim_{x\to 0} \frac{3-(x+3)}{3x(x+3)} = \lim_{x\to 0} \frac{3-x-3}{3x(x+3)} = \lim_{x\to 0} \frac{-x}{3x(x+3)} = \lim_{x\to 0} \frac{-1}{3(x+3)} = \frac{-1}{3(0+3)} = \frac{-1}{9}$

(t) $\lim_{x\to\infty} \frac{7x-8x}{8x^2+6x+5} = \lim_{x\to\infty} \frac{\frac{4x-8x}{x^2}}{\frac{8x^2+6x+5}{x^2}} = \lim_{x\to\infty} \frac{4-\frac{8}{x}}{8+\frac{6}{x}+\frac{5}{x^2}} = \frac{4-0}{8+0+0} = \frac{4}{8} = \frac{1}{2}$

(u) $\lim_{t\to 0} \frac{\sin 3t}{\sin 5t} = \lim_{t\to 0} \frac{3}{3}\cdot\frac{5}{5}\cdot\frac{\sin 3t}{1}\cdot\frac{1}{\sin 5t} = \lim_{t\to 0} \frac{3}{5}\cdot\frac{\sin(3t)}{3}\cdot\frac{5}{\sin 5t} = \frac{3}{5}\cdot 1\cdot 1 = \frac{3}{5}$

(v) $= \lim_{x\to 0} \frac{\sin(3x)}{\frac{x}{3}} = \lim_{x\to 0} \sin(3x)\cdot\frac{3}{x} = 3\lim_{x\to 0}\frac{\sin(3x)}{x} = 3\cdot\lim_{x\to 0}\frac{3}{3}\cdot\frac{\sin(3x)}{x}$

$= 9\lim_{x\to 0}\frac{\sin(3x)}{3} = 9\cdot 1 = 9$

(w) $\lim_{x\to 3} f(x)$ $\quad f(x) = \begin{cases} x^2 \text{ if } x\le 3 \\ 9-x \text{ if } x>3 \end{cases}$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 9-x = 9-3 = 6$ > Not equal

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} x^2 = 3^2 = 9$

$\lim_{x\to 3} f(x) =$ DNE

2. Given $\lim_{x\to c} f(x) = -\frac{1}{2}$ and $\lim_{x\to c} g(x) = \frac{2}{3}$

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \frac{-\frac{1}{2}}{\frac{2}{3}} = -\frac{1}{2}\cdot\frac{3}{2} = -\frac{3}{4}$$

3. $|f(x)-L| = |2x-1-3| = |2x-4| = 2|x-2| < \frac{\epsilon}{2}$ Let $\delta = \frac{\epsilon}{2}$

$|f(x)-L| = |2x-1-3| = |2x-4| = 2|x-2| < 2\cdot\frac{\delta}{2} = < \delta$