MAC 2311
Test 4 Review

1. For the following functions, find *all* antiderivatives:

a) $f(x) = x^5$ b) $f(x) = 3x^2 - \frac{2}{x^2}$

c) $f(x) = \cos(4x)$ d) $f(x) = \sqrt[3]{x} + \sec^2(x) + \sec(x)\tan(x)$

2. Evaluate each of the following integrals:

a) $\int \left(4x^5 - 3x^3 + \frac{1}{2}x - 1\right) dx$ b) $\int \left(x^3 \sqrt[3]{x}\right) dx$

c) $\int (\sec^2 x - \csc^2 x) dx$ d) $\int \frac{x^3 - 2x^2 + 1}{\sqrt{x}} dx$

e) $\int 3\left(4x^2 - \sin x\right) dx$ f) $\int \left(\csc x + \cot x\right)^2 dx$

g) $\int (x-1)^2 (x+4) dx$ h) $\int x^2 \left(2x^2 - \frac{2}{x}\right) dx$

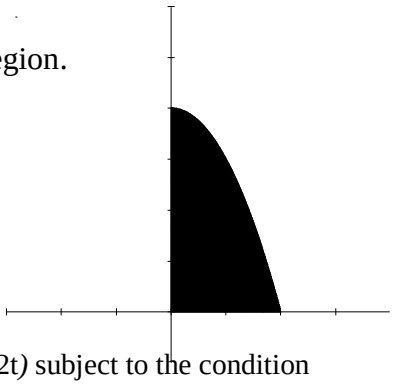
i) $\int \frac{\sqrt[3]{x}}{x^2} dx$ j) $\int \frac{x^2 + x + 1}{\sqrt{x}} dx$

3. For the following function: $f(x) = (x-1)^2 + 3$ from [0, 4], using 4 subintervals, approximate the area using a) The lower (left) Riemann Sum b) The upper (right) Riemann Sum

4. Use the properties of sigma notation and the summation formulas to evaluate the sum:
$\sum_{i=1}^{10} (i^2 + 3i - 2).$

5. Let $s(n) = \sum_{i=1}^{n} \left(1 + \frac{i}{n}\right)^2 \left(\frac{1}{n}\right)$. Find the limit of $s(n)$ as $n \to \infty$.

6. Find the definite integral, which represents the area of the shaded region.



7. Solve the differential equation; $f''(x) = x^2$, $f'(0) = 6$, $f(0) = 3$

8. A particle moves on the *x*-axis so that at any time *t* its velocity v (t) =sin(2t) subject to the condition *x (0) = 0* where *x (t)* is the position function. Write an expression for *x (t).*

9. Find the value(s) of c guaranteed by the MVT for integrals of $f(x) = \sec^2(x) on [-\pi/4, \pi/4]$

10. Evaluate the integral: $\int_2^4 |x-3|dx$

11. For what value of *k*, *k* > *0*, does $\int_0^k (4kx-5k)dx = k^2$?

12. The velocity of a function is v (t) =-2cos3t. Find the position function if $s\left(\frac{\pi}{2}\right)=2$

13. The acceleration of a particle is given by the equation a (t) =2 t -1. At 3 seconds, the velocity is 2 and the position is 4.

    a. The velocity function is _____b. The position function is: _____

14. Use the Fundamental Theorem of Calculus to simplify the expression $\frac{d}{dx}\int_x^{x^3} \sin(t^2)dt$

15. Find the particular solution of the differential equation $f''(x)=4(x+2)$ whose graph passes through the point (0, 3) and for which $f'(1)=4$

16. If $F(x)=\int_0^x \sqrt{t^3+1}dt$, find $F'(2)$

17. If $\int_d^b f(x)dx = a+2b$, find $\int_a^b (f(x)+5)dx$

18. Let f be a differentiable function for all *x*. Which of the following must be true?

I. $\frac{d}{dx}\int_0^3 f(x)dx = f(x)$ II. $\int_3^x f'(x)dx = f(x)$ III. $\frac{d}{dx}\int_3^x f(x)dx = f(x)$

19. Evaluate each of the following integrals:

a. $\int \left(\sec^2 3x\right)dx$ b. $\int 5x\left(4x^2-2\right)^5 dx$ c. $\int 3x\left(6x^2-2\right)^5 dx$

d. $\int_0^4 \frac{5}{3x+1}dx$ e. $\int\left(\frac{(x+1)}{\sqrt{x^2+2x-4}}\right)dx$ f. $\int\left(\frac{x}{\left(x^2+4\right)^3}\right)dx$

g. $\int \sec^2 x \tan^2 x dx$ h. $\int \cos\frac{x}{4}dx$ i. $\int_0^{\pi/3}\frac{\sin(x)}{\cos^2(x)}dx$

j. $\int (x+3)^3(x-5)dx$ k. $\int \frac{1}{\theta^2}\cos\frac{1}{\theta}d\theta$ l. $\int e^{\tan 2x}\sec^2 2x dx$

m. $\int_{-2}^{4} x^2(x^3+8)^2\,dx$ n. $\int_{0}^{2} \frac{x}{\sqrt{1+2x^2}}\,dx$ o. $\int \frac{x}{x^2+1}\,dx$

p. $\int_{e}^{e^4} \frac{1}{x\sqrt{\ln x}}\,dx$ q. $\int_{0}^{\frac{\pi}{4}} \tan\left(\frac{\pi}{4}-x\right)dx$ r. $\int_{0}^{\pi} \cos\frac{x}{2}\,dx$

s. $\int_{0}^{1} xe^{-x^2}\,dx$ t. $\int (x^2+2)^2 \cdot 2x\,dx$ u. $\int (2x^3+5)^3 \cdot 24x^2\,dx$

v. $\int (3x^5+1)^6 \cdot x^4\,dx$ w. $\int 2\cos(2x)dx$ x. $\int x^3(1-x^4)^5\,dx$

y. $\int x^2\sqrt{x^3+1}\,dx$ z. $\int \frac{\cos(\sqrt{x})}{\sqrt{x}}dx$

20. Find the area enclosed by the graphs of $y = 4x - x^2$ and $y = -2x + 5$.

21. Find the area of the region lying between $y = -x^2 + 7x - 10$ and the x-axis.

22. Find the area enclosed by the graphs of $y = \sqrt{x}$, $y = 4 - \frac{1}{2}x$ and $y = 0$.

23. Find the area enclosed between the x-axis and the curve $y = 4 + 3x - x^2$.

24. Evaluate the following integrals.

a. $\int_{3}^{8} \frac{1}{x}\,dx$ b. $\int \frac{7x^2}{4x^3+5}\,dx$ c. $\int_{5}^{7} \frac{t}{9-t^2}\,dt$

d. $\int \frac{4t^3-5t^2+6}{8t}\,dt$ e. $\int_{-6}^{-2} \frac{5-7x}{3x^2}\,dx$

f. $\int \frac{dw}{w(\ln w)^3}$ g. $\int_{4}^{64} \frac{dt}{6t\sqrt[4]{\log_4 t}}$

h. $\int \frac{\csc^2\theta}{8+3\cot\theta}\,d\theta$ i. $\int \frac{4e^{2x}}{e^{2x}-4}\,dx$

25. Integrate:

a. $\int_{1}^{9} \frac{1}{2x}\,dx$ b. $\int_{\pi}^{2\pi} \cos\theta\,d\theta$ c. $\int_{1}^{2} \frac{3}{t^4}\,dt$ d. $\int x(1+2x^4)dx$

e. $\int \left(2-\sqrt{x}\right)^2 dx$ f. $\int (\sin x)^3 \cos x\,dx$ g. $\int_{1}^{4} \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx$

26. Find the area under the curve $y = x^4 + 2$ from x = 1 to x = 3.

27. Find the area of the region enclosed by $y = \frac{1}{x}, \quad y = \frac{1}{x^2}, \quad x = 2$

28. Find the area of the region enclosed by the curves $x = 1 - y^2, \quad x = y^2 - 1$

Answers:

1. a. $\frac{x^6}{6} + c$ b. $x^3 + \frac{2}{x} + c$ c. $\frac{\sin(4x)}{4} + c$

   d. $\frac{4}{3}x^{4/3} + \tan(x) + \sec(x) + c$

2. a. $\frac{2x^6}{3} - \frac{3}{4}x^4 + \frac{1}{4}x^2 + c$ b. $\frac{3}{13}x^{13/3} + c$ c. tan(x) + cot (x) + c

   d. $\frac{2}{35}\sqrt{x}(5x - 14)x^2 + 35)$ e. $4x^3 + 3\cos(x) + c$

   f. -x -2 cot (x) -2csc(x) + c g. $\frac{1}{12}x(3x^3 + 8x^2 - 42x + 48) + c$

   h. $\frac{2x^5}{5} - x^2 + c$ i. $-\frac{3}{2}x^{-2/3} + c$

   j. $\frac{2}{15}x^{1/2}(3x^2 + 5x + 15)$

3. a) 18 b) 26

4. 530 5. 7/3 6. 16/3

7. $f(x) = \frac{x^4}{12} + 6x + 3$ 8. $x(t) = -\frac{1}{2}\cos(2t) + \frac{1}{2}$ 9. 0.48

10. 1 11. 3 12. $s(t) = -\frac{2}{3}\sin(3t) + \frac{4}{3},$

13. $a) v(t) = t^2 - t - 4$

    $b) s(t) = \frac{t^3}{2} - 0.5t^2 - 4t + 11.5$

14. $-\sin x^2 + 3x^2 \sin x^6$ 15. $\frac{2}{3}x^3 + 4x^2 - 6x + 3$

16. 3 17. *7b-4a* 18. II & III

19. a. $\frac{\tan(3x)}{3} + c$ b. $\frac{5}{48}(4x^2 - 2)^6 + c$ c. $\frac{1}{24}(6x^2 - 2)^6 + c$

d. $\frac{5}{3}\ln(13)$ e. $\sqrt{x^2+2x-4}+c$ f. $\frac{-1}{4(x^2+4)^2}+c$

g. $\frac{\tan^3(x)}{3}+c$ h. $4\sin(\frac{x}{4})+c$ i. 1

j. $\frac{x^5}{5}+x^4-6x^3-54x^2-135x+c$ k. $-\sin(\frac{1}{\theta})+c$

l. $\frac{1}{2}e^{\tan(2x)}+c$ m. 41472 n. 1

o. $\frac{1}{2}\ln\left|x^2+1\right|+c$ p. .693 q. 0.346

r. 2 s. $\frac{-1}{2e}=.316$ t. $\frac{(x^2+2)^3}{3}+c$

u. $(2x^3+5)^4+c$ v. $\frac{1}{105}(3x^5+1)^7+c$ w. sin (2x) + c

x. $\frac{-(1-x^4)^6}{24}+c$ y. $\frac{2}{9}(x^3+1)^{3/2}+c$ z. $2\sin(\sqrt{x})+c$

20. 32/3 21. 9/2 22. 28/3 23. 125/6

24. a. $\ln\frac{8}{3}$ b. $\frac{7}{12}\ln\left|4x^3+5\right|+c$

c. $\frac{1}{2}\ln\frac{2}{5}$ d. $\frac{1}{8}\left[\frac{4t^3}{3}-\frac{5t^2}{2}+6\ln\left|t\right|\right]+c$

e. $\frac{5+21\ln 3}{9}$ f. $-\frac{1}{2(\ln w)^2}+c$

g. $\frac{(\sqrt[4]{27}-1)\ln 16}{9}$ h. $-\frac{1}{3}\ln\left|8+3\cot\theta\right|+c$

i. $\ln(e^{2x}-4)^2+c$

25.a. ln 3 b. 0 c. 7/8 d. $\frac{x^2}{2}+\frac{x^6}{3}+C$

e. $4x-\frac{8}{3}x^{3/2}+\frac{1}{2}x^2+C$ f. $\frac{\sin^4 x}{4}+C$ g. $\frac{4\sqrt{2}}{3}-\frac{5\sqrt{5}}{12}$

26. 262/5 27. ln (2) -1/2 28. 8/3