MAC 2311 **Test 3 Review**

Textbook chapter 4 review p.274

1, 5, 9, 11, 13, 17, 23, 27, 31, 33, 35, 37, 41, 43, 45, 47, 55, 59, 60, 73, 79, 95.

1. Find all the critical numbers for a) $f(x) = \frac{x-1}{x+1}$ b) $f(x) = x\sqrt{2x+1}$.

   a) none **b) -1/2, -1/3**

2. Find the absolute extrema

   a) on [-1, 2] of $f(x) = \frac{10}{x^2+1}$. **a.max (0, 10), a.min (2, 2)**

   b) on [0, 2π] for y = sin x + cos x **a.max** $(\frac{\pi}{4}, \sqrt{2})$ **; a.min** $(\frac{5\pi}{4}, -\sqrt{2})$

3. Find the c guaranteed by the mean value theorem on the interval [2, 3] for $f(x) = 3x - x^2$

   c = 2.5

4. Find all the open intervals on which $f(x) = \frac{x}{x^2+x-2}$ increasing or decreasing.

   Dec (-∞, -2), (1, ∞), Inc (-2, 1)

5. Find the x values that give relative extrema for a) $f(x) = 3x^5 - 5x^3$. b) $y = \frac{2x}{(x+4)^3}$.

   c) $f(x) = x^{\frac{1}{3}}(8-x)$ **a) r. min x = 1, r. max x = -1**

   c) r. max at x = 2.

6. Find the intervals on which the graph of the function $f(x) = x^4 - 4x^3 + 2$ is concave upward or downward. Find the points of inflection.

   $(-\infty, 0), (2, \infty)$ *c.up*
   $(0, 2)$ *c.down*
   I.Point $(0, 2), (2, -14)$

7. Find all of the points of inflection of the graph of

   a) $f(x) = x^4 - x^3$. b) $f(x) = 2x(x-4)^3$.

   a) (0, 0), (0.5, -.625 **b) (4, 0), (2, -32)**

8. a) Let $f(x) = x^3 - x^2 + 3$. Use the second derivative test to find the relative extrema.

   b) Let $f(x) = x^4 + 4x^3$. Use the second derivative test to find the relative extrema.

   a) r. min. (2/3, 77/27 b) r.min (-3, -27)

9. Use the techniques learned in this chapter to sketch $f(x) = x^3 + x^2 - 6x$

10. Determine whether Rolle's Theorem can be applied to the function, $f(x)=(x-2)(x+3)^2$ on the interval [–3, 2]. If it can, then find all values of *c* guaranteed by the theorem.

c=1/3, c=-3

11. Determine whether the Mean Value Theorem can be applied to the function, $g(x)=x^{\frac{2}{3}}$ on the interval [1, 8]. If it can, then find all values of *c* guaranteed by the theorem.

c=3.76

12. For $0 \le t \le 4$ a particle moves along the *x*-axis. The velocity of the particle at any time *t* is given by $v(t)=\cos\left(\frac{\pi}{2}t\right)$.

a. Find the acceleration at any time *t*' at t = 3. $\frac{-\pi}{2}\sin(\frac{\pi}{2}t);\frac{\pi}{2}$

b. Is the velocity increasing, decreasing or neither at *t* = 3. Explain **Increasing** $v'(3)>0$

13. Use the differential to approximate $\sqrt{82}$ **(163/18 = 9.05)**

14. . For $y=\frac{x^3}{3}-\frac{x^2}{2}-2x$ on [- 2, 3] find *c* satisfying Rolle's Theorem. **Does not apply.**

15. For $y=x^2-2x$ on [- 3, 1] find *c* satisfying the Mean Value Theorem. ***c* = - 1**

16. Determine where the function $f(x)=x^4-2x^2$ is increasing and decreasing and find all relative extrema, if any. **r.max (0, 0), r. min (-1, -1), 1, -1)**

17. If $f(x)=x^3+x^2-5x-5$, determine intervals on which the graph of *f* is concave up and intervals on which the graph is concave down. c.up $(-1/3,\infty)$, c.down ($(-\infty,-1/3)$

18. Find the differential of $f(x)=4x^{5/2}-x^{-1/2}$ $(10x^{3/2}+(1/2)x^{-3/2})dx$

19. Find the interval where $f(x)=1-x^{\frac{1}{3}}$ is concave up, if any. $(0,\infty)$

20. Graph the function $f(x)=\frac{x^2}{x^2-1}$. Note that $f'(x)=\frac{-2x}{\left(x^2-1\right)^2}$ and $f''(x)=\frac{2\left(3x^2+1\right)}{\left(x^2-1\right)^3}$.

21. Find the total number of local maximum or minimum points on $f'(x)=x(x-3)^2$.

Min @ *x* = 0

22. If $f(x) = 2x(x-1)^2$, when is the graph of *f* concave down? **(-∞, 2/3)**

23. Find where $y = 5x^3 - 20x$ any relative maximum or minimum values has.

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Max @ $-\frac{2}{\sqrt{3}}$**, min @** $\frac{2}{\sqrt{3}}$

24. $y = x^4 - 4x^3$, Find the total number of inflection points. **@ *x* = 0, 2**

25. The derivative of a function *f* is given for all *x* by $f'(x) = x^2(x+1)^3(x-4)^2$. Find all *x* values where a) the critical points occur b) *f* is increasing or decreasing

a) – 1, 0, 4 b) (-∞, -1) dec; (-1, 0), (0, 4), (4, ∞) inc.

26. A farmer wants to fence in a piece of land that borders on one side on a river. She has 200m of fence available and wants to get a rectangular piece of fenced-in land. One side of the property needs no fence because of the river. Find the dimensions of the rectangle that yields maximum area.

50x100

27. An open box with a rectangular base is to be constructed from a rectangular piece of cardboard 16 inches wide and 21 inches long by cutting out a square from each corner and then bending up the sides. Find the size of the corner square which will produce a box having the largest possible volume. **x = 3**

28. The product of two positive numbers is 588. Minimize the sum of the first and three times the second. **14; 42**

29. a. Find the local linear approximation of f(x) = $3x^2 - 4x$ at the point where x = 3.

b. Use your approximation to estimate f (2.9), and f (3.1).

a) y = 14x -27 b) 13.6; 16.4

30. a) Find d*y* and Δ*y* for f(x) = $3x^2 + 4x$ at *x* = -2 and d*x* = Δ*x* = .01.

b) Find d*y* and Δ*y* for f(x) = x^3 -2 x at *x* = 2 and d*x* = Δ*x* = .1

a**)** **dy =Δ*y*=-0.08** **b) dy =1, Δ*y*=*1.061***

31. The curve $2x^2y + y^2 = 2x + 13$ passes through (3, 1). Use the tangent line to the curve to find the approximate value of *y* at *x* = 2.8.

y = (-1/2) x + (5/2) *f* (2.8) = 1.1