MAC 2311
Test 2 / Chapter 3

Textbook p.196-198 #3, 5, 13, 19, 23, 29-47 (e o o), 49, 55-83 (e o o), 85, 93, 97, 99, 105, 109.

1. Use the definition of a derivative only to calculate the derivative of

a. $f(x) = x^2 - 3x + 1$ b. $y = \dfrac{1}{x-1}$ c. $f(x) = \sqrt{x}$

2. Differentiate: a. $y = \dfrac{2x}{1-3x^2}$. b. $y = x^3\sqrt{2x+1}$

c. $y = \cot^3 \sqrt{x}$. d. y = cos $^{-1}e^{3x.}$ e. $y = \dfrac{x}{x+y}$

f. $f(\theta) = \sec(\theta^2)$ g. $y = \sin^2 x - \cos^2 x$. h. $f(x) = 2x^3 - 7x + 5$

i. $y = (2x^{-8} + 5)(5x^6 + 7)$ j. $y = (3x^2 + 7x)(12x^{-4} + 2x^5)$

k. $f(x) = e^{x^2} \sin^{-1}(2x)$ l. $f(x) = \dfrac{\ln(3x-2)}{x^2+1}$ m. $f(x) = \tan(2x)$

n. $y = xe^{3x-2}$ o. $f(x) = \tan^{-1}\left(\sqrt{x}\right)$ p. $f(x) = 7^{x^2-2}$

q. $f(x) = \sin^{-1}(x^2 + 1)$

3. Find: a. $\dfrac{d}{dx}\left[\sin\left(x^2 - 1\right)\right]$ b. $\dfrac{d}{dx}\left[\ln\left(\ln\left(x^2\right)\right)\right]$

c. $\dfrac{d}{dx}\left[\left(\ln\left(\ln x\right)\right)^2\right]$ d. $\dfrac{d}{dx}\left[x^2 - \dfrac{2}{x}\right]$ e. $\dfrac{d}{dx}(\dfrac{e^x+1}{\sqrt{x}})$

f. $\left.\dfrac{df}{dx}\right]_{x=2}$ if $f(x) = \dfrac{x+1}{x-1}$ g. $\dfrac{d^2y}{dx^2}$ if $y = x\sin(x)$ h. $x = \cos y$

4. Find the equation for the line tangent to $x\cos(y) + x^2 - 2y = 0$ at the point (0, 0)

5. Using implicit differentiation, find y'' $x^2 + y^2 - 2xy = 1$

6. The position function for the movement of a particle is given by $s = (t^3 + 1)^2$ where s is measured in feet and t is measured in seconds. Find the acceleration of the particle at 1 second.

7. Find an equation for the tangent line to the graph of $f(x) = \sqrt{x+1}$ at the point where x = 3.

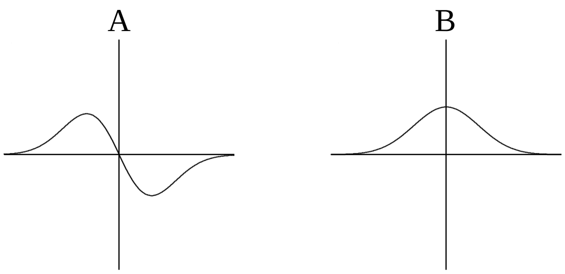
8. Find the values of x for all points on the graph of $f(x) = x^3 - 2x^2 + 5x - 16$ at which the slope of the tangent lie is 4.

9. Given the function $R = 2x^2 + \dfrac{1}{x}$, compare the average rate of change from x = 1

to $x = 3$ with the instantaneous rate of change when $x = 2$.

10. A kite is flying 150 m high, where the wind causes it to move horizontally at the rate of 5 m per second. In order to maintain the kite at a height of 150 m, the person must allow more string to be let out. At what rate is the string being let out when the length of the string already out is 250 m?

11. Two graphs are drawn below. Determine which is f and which is f'.



12. Use implicit differentiation to find an equation of the tangent line to the curve $x^2 - y^2 = 2xy - x$ at the point $(1,1)$

13. As a balloon in the shape of a sphere is being blown up, the volume increases at a rate of 4 cubic inches per second. At what rate is the radius increasing when the radius is 1 inch?

14. Find an equation of tangent line to the curves $f(x) = x^2 - x$ at $a = -1$

15. Find an equation of the tangent line to the curve $f(x) = \sin x$ at $a = \frac{3\pi}{4}$. Use exact values, not decimal approximations.

16. A model rocket is fired straight up from the ground, and its height in meters t seconds after it is fired is described by the curve $h(t) = -4.9t^2 + 90t$.
    a. What is the average velocity from the start until it reaches its highest point?
    b. What is the instantaneous velocity 3 seconds after it is fired?

17. A ladder 16 feet long is resting against a vertical wall. If the angle between the top of the ladder and the wall is changing at a constant rate of 1/2 radian per minute, how fast is bottom of the ladder sliding away from the wall when the angle is $\pi/6$ radians?

18. A ladder 13 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at the rate of 0.5 ft/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 5 feet from the wall? At what rate is the angle between the ladder and the ground changing?

19. A car is heading east toward an intersection at the rate of 40 mph. A truck is heading south, away from the same intersection at the rate of 60 mph. At what rate is the distance between the car and the truck changing when the car is 8 miles from the intersection and the truck is 15 from the intersection?

20. A cylindrical tank of water has a height of 6 feet and its radius is one third of its height. Water is flowing into the tank at the rate of 10 ft^3/min. How fast is the height of the water level in the tank rising?

21. If $f(x)=\log_2(3x)$, then; $f'(x)$ equals

a) $\frac{1}{3x}$ b) $\ln(3x)$ c) $\frac{1}{x\ln 2}$

d) e^{3x} e) $2e^{3x}$

22. Findy'', if $y=x^6-\frac{x^5}{5}+x^2-2$

a) $y''=6x^5-x^4+2x$ b) $y''=30x^5-4x^3+2$

c) $y''=120x^3-12x^3$ d) $y''=30x^4-4x^3$ e) None of these

23. Find the slope of the tangent to the curve $x^2y+3x^2y^3+4=0$ at the point $(1,-1)$

a) 0.4 b) 0.8

c) 1.0 d) 1.6 e) 2.0

24. Write an expression for the tangent line to $f(x)=x^3$ at $x=2$.

a) $y-8=12(x-2)$ b) $y-8=8(x-2)$

c) $y-2=12(x-8)$ d) $y-8=\frac{-1}{12}(x-2)$ e) None of these

ANSWERS:

1. a) $\frac{-1}{x^2}$ b) $\frac{-1}{(x-1)^2}$ c) $\frac{1}{2\sqrt{x}}$

2.

a) $\frac{6x^2+2}{(1-3x^2)^2}$ b) $\frac{7x^3+3x^2}{\sqrt{2x+1}}$ c) $\frac{-3}{2\sqrt{x}}\cot^2\sqrt{x}\csc^2\sqrt{x}$

$d)\frac{-3e^{3x}}{\sqrt{1-e^{2x}}}, e)\frac{y}{1+(x+y)^2}, f)2\theta\sec^2\theta\tan^2\theta, g)2\sin(2x)$

$h)6x^2-7, i)\frac{-20}{x^3}-\frac{11^2}{x^9}+150x^5$

j) $y=(3x^2+7x)(12x^{-4}+2x^5)$

$k)\frac{2e^{x^2}}{\sqrt{1-4x^2}}+\sin^{-1}(2x)2xe^{x^2}, l)\frac{(x^2+1)(\frac{3}{3x-2})-(2x)(\ln(3x-2)}{(x^2+1)^2}$

$m)2\sec^2(2x), n)e^{3x-2}(3x+1), o)\frac{1}{2\sqrt{x(1+x)}}$

$p)7^{x^2-1}2x\ln(7), q)\frac{2\sqrt{x}}{\sqrt{x+2}}$

3. $a) 2x\cos(x^2-1), b) \frac{2}{x\ln x^2}, c) \frac{1}{2x\ln x}, d) 2x+\frac{2}{x^2}$
$e) \frac{2xe^x - e^x + 1}{2x^{3/2}}, f) -2, g) x\cos x + \sin x, h) -\csc y$

4. y = (1/2) x 5. $y'' = 0$ 6. 42 7. $y = \frac{1}{4}x + \frac{5}{4}$

8. 1/3, 1 9. 7.66, 7.75 10. 4 m/sec

11. A: First Derivative, B: Function 12. $y = \frac{1}{4}x + \frac{3}{4}$ 13. $\frac{1}{\pi} inch/\sec$

14. $y = -3x - 1$ 15. $-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{8}(3\pi + 1)$

16. a) 45 b) 60.6 17. $4\sqrt{3}$ 18. -5/24

19. 71.76 20. $\frac{5}{6\pi}$ 21. C 22. e

23. b 24 a