MAC 2311
Test 1 Review

1. Find each of the following limits or state that the limit does not exist.

1. $\lim_{\Delta x \to 0} \dfrac{\frac{1}{(3+\Delta x)^2} - \frac{1}{9}}{\Delta x}$

2. $\lim_{x \to 0} \dfrac{3 - \sqrt{x+9}}{x}$

3. $\lim_{\Delta x \to 0} \dfrac{(2+\Delta x)^3 - 8}{\Delta x}$

4. $\lim_{x \to 0} \dfrac{\sin 4x}{x}$

5. $\lim_{x \to 0} \dfrac{\tan 3x}{\sin 5x}$

6. $\lim_{x \to 0} \dfrac{1 - \cos x}{x \cos x}$

7. $\lim_{x \to 2} \frac{1}{x-2}$.

8. $\lim_{x \to 3} \left[4x^3 + 3x^2 - 2x + 1 \right]$

9. $\lim_{x \to 3} \dfrac{2x-3}{x+5}$

10. $\lim_{x \to -5} \dfrac{2x-3}{x+5}$

11. $\lim_{x \to 4} g\left(f(x)\right)$, if $f(x) = 2x^2 - 3x + 1$, $g(x) = \sqrt[3]{x+6}$

12. $\lim_{x \to 2} \left(\sqrt{x-3} \right)$

13. $\lim_{t \to 0} \dfrac{\sqrt{t+2}\sin(t)}{t}$

14. $\lim_{t \to 0} \dfrac{\cos(t) - \cos^2(t)}{t}$

15. $\lim_{x \to 4} \dfrac{4-x}{\sqrt{x}-2}$

16. $\lim_{x \to \pi} \dfrac{\pi x - \pi^2}{2x - 2\pi}$

17. $\lim_{x \to 0} \dfrac{\tan(x)}{x}$

18. $\lim_{x \to 3} \dfrac{(9-x^2)}{x-3}$

19. $\lim_{x \to \infty} \sqrt{\dfrac{5x^2-2}{2x+3x^2}}$

20. $\lim_{x \to -\infty} \dfrac{-1}{2x}$

21. $\lim_{x \to \infty} \dfrac{x-2}{\sqrt{x^2+7}}$

22. $\lim_{t \to -2} \dfrac{2t+4}{12-3t^2}$

23. $\lim_{x \to 0^+} \dfrac{1}{x^2}$

24. $\lim_{x \to -4^-} \dfrac{x}{x+4}$

25. $\lim_{x \to 0} \dfrac{\sin(x)\cos(x)}{x}$

26. $\lim_{x \to \frac{\pi}{2}^-} \sin x$

27. $\lim_{x \to 3^+} \dfrac{x^2+3x}{x^2-9}$

28. $\lim_{x \to -1} \left(x^3 + \sqrt{x^2-1} + x \right)$

29. $\lim_{x \to -1^-} \dfrac{1}{x+1}$

30. $\lim_{x \to 2} \dfrac{3x^2-4x-4}{2x^2-8}$

31. $\lim_{x \to k} \dfrac{x^2-k^2}{k^4-x^4}$

32. $\lim_{t \to 0} \dfrac{(t^2-3)^2-9}{t^2}$

33. $\lim_{x \to 4} \sin^{-1}(\frac{x}{4})$

34. $\lim_{x \to \infty} 2 + e^{-x}$

35. $\lim_{Vx \to 3^+} \dfrac{(Vx)^3 - 27}{Vx - 3}$

36. $\lim_{x \to -3} \dfrac{|x+3|}{x+3}$

37. $\lim_{x \to \infty} \dfrac{1-x}{3x^2-7x+4}$

38. $\lim_{x \to 2} \left(\dfrac{7x+2}{4-x} \right)^{2/3}$

39. $\lim_{x \to 1} e^{(x^2-x)}$

40. $\lim_{x \to \frac{\pi}{2}} 3^{\sin(x)}$

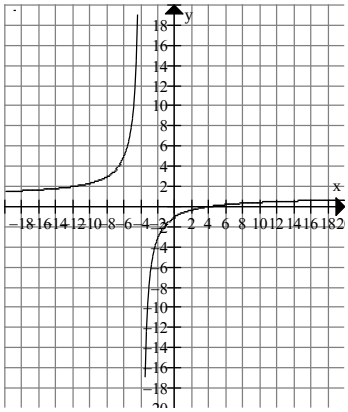
2. Are the functions continuous on the given intervals? Explain, why or why not?

a. $\dfrac{1}{\cos(x)}$ *on* $[0, \pi]$

b. $\dfrac{e^x}{e^x - 1}$ *on* $[-1, 1]$

c. $\dfrac{1}{x-2}$ *on* $[-1, 1]$

3. Evaluate the limits from the following graph of g(x).



a) $\lim_{x\to+\infty} g(x)$ b) $\lim_{x\to-4^+} g(x)$

c) $\lim_{x\to-\infty} g(x)$ d) $\lim_{x\to-4^-} g(x)$

e) $\lim_{x\to-4} g(x)$

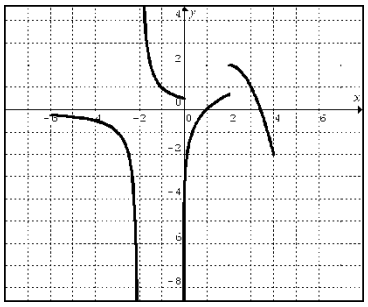
4. using a table, evaluate

$\lim_{x\to-2} (x^2 - 2x)$

x	*1.923*	*1.95*	*1.999*	*2*	*2.001*	*2.10*	*2.15*
f(x)							

5. Use the intermediate value theorem to prove that the polynomial $x^5 + 4x^2 - 2x + 3$ has at least one real zero between -2 and -1.

6. Find each limit. Note: the points (0, ½) and (2, 2) are not on the graph of the function f. Any vertical asymptotes?



(a) $\lim_{x\to0^+} f(x)$ (b) $\lim_{x\to2^+} f(x)$

(c) $\lim_{x\to2^-} f(x)$ (d) $\lim_{x\to-2^+} f(x)$ (e) $\lim_{x\to1} f(x)$

7. Let $\lim_{x\to-1} f(x) = 2$, $\lim_{x\to-1} g(x) = -3$, *and* $\lim_{x\to-1} h(x) = 4$ and find each limit:

(a) $\lim_{x\to-1}\left[2f(x) - 3h(x)\right]$ (b) $\lim_{x\to-1}\sqrt{f(x)h(x)}$

8. Which of the functions is continuous at $x = 5$? Explain.

(a) $f(x) = \dfrac{1}{x}$ (b) $f(x) = \dfrac{1}{5-x}$ (c) $f(x) = \begin{cases} \dfrac{x^2-25}{x-5}, & x \neq 5 \\ 10, & x = 5 \end{cases}$

9. Determine if the function is continuous at the given value of a.

(a) $f(x)=\begin{cases} x^2, & x>1 \\ -x+2, & x\le 1 \end{cases}$ $a=1$ (b) $f(x)=\begin{cases} -x^3+3, & x\le 0 \\ \sqrt{x}, & x>0 \end{cases}$ $a=2$

10. Evaluate the limit assuming that $\lim_{x\to -4} f(x)=3$ and $\lim_{x\to -4} g(x)=1.$

a. $\lim_{x\to -4} f(x)g(x)$ b. $\lim_{x\to -4}(2f(x)+3g(x))$

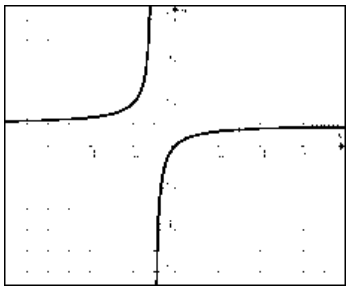
c. $\lim_{x\to -4} \frac{g(x)}{x^2}$ d. $\lim_{x\to -4} \frac{f(x)+1}{3g(x)-9}$

11. Determine the value of a, and b, so that each function is continuous.

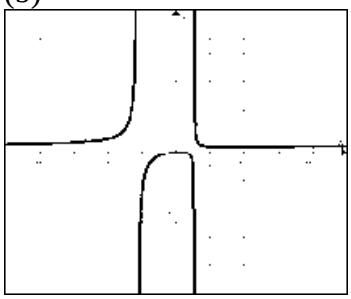
(a) $f(x)=\begin{cases} ax+4, & x<1 \\ x^2-2a, & x\ge 1 \end{cases}$ (b) $f(x)=\begin{cases} x^2-bx, & x<1 \\ ax, & 1\le x<2 \\ ax^3+b, & x\ge 2 \end{cases}$

12. Use the graphs to write the horizontal asympotes using limits.

(a)



(b)



13. Find the horizontal asymptote of each function if any.

(a) $\frac{2x^2}{4-x^2}$ (b) $\frac{x}{x^3+6x^2+x}$ (c) $\frac{100e^{2t}-t}{2-e^{2t}}$

(d) $\frac{1}{2+e^{-t}}$ (e) $\frac{x^4-1}{x^2+1}$

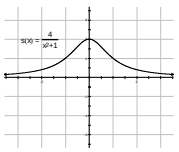
14. Use the definition of the limit to show that

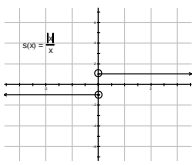
a) $\lim_{x\to -1}(4x-1)=3$

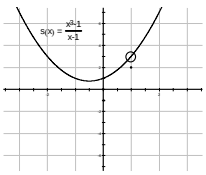
b) $\lim_{x\to -2}(3x+5)=-1$

15. Determine the intervals where the following functions are continuous.

(a) (b) (c)

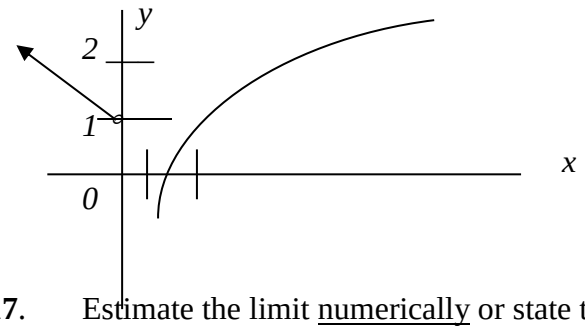






16. The function *f(x)* is sketched below. (0, 1) is not on the graph. Find:

a) $\lim_{x\to 0^-} f(x)$ b) $\lim_{x\to 0^+} f(x)$ c) $\lim_{x\to 0} f(x)$



17. Estimate the limit numerically or state that the limit does not exist.

1. $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$ 2. $\lim_{x\to 3} \frac{x-3}{x^2-10x+21}$

18. Fill in the table and guess the value of the limit.

$$\lim_{x\to 2} f(x), \text{ where } f(x) = \frac{x^2-x-2}{x^2+x-6}.$$

x	f(x)		x	f(x)
2.002			1.998	
2.001			1.999	
2.0001			1.9999	

19. Graph the function and use the graph to estimate the value of the limit.

a. $\lim_{\theta\to 0} \frac{\sin(3\theta)}{\sin(2\theta)}$ b. $\lim_{x\to 0} \frac{2^x-1}{4^x-1}$

Answers:

1.

1.	-2/27	2.	-1/6	3.	12	4.	4
5.	3/5	6.	0	7.	DNE	8.	130
9.	3/8	10.	DNE	11.	3	12.	DNE
13.	$\sqrt{2}$	14.	0	15.	-4	16.	$\frac{\pi}{2}$
17.	1	18.	-6	19.	$\frac{\sqrt{15}}{3}$	20.	0

21. 1 22. 1/6 23. ∞ 24. ∞

25. 1 26. 1 27. ∞ 28. 2

29. $-\infty$ 30. 1 31. $\frac{-1}{2k^2}$ 32. -6

33. $\frac{\pi}{2}$ 34. 2 35. 27 36. DNE

37. 0 38. 4 39. 1 40. 3

2. a) No b) No c) Yes

3. a) 1 b) $-\infty$ c) 1 d) ∞ e) DNE

4. 0

6. a) $-\infty$ b) 2 c) 1/2 d) ∞ e) 0

7. a) -8 b) $2\sqrt{2}$ 8.a, c

9. a) Yes b) No

10. a) 3 b) 9 c) 1/16 d) -2/3

11. a) -1 b) a = -1/5, b = 6/5

12. a) y = 1 b) y = 0

13. a) -2 b) 0 c) -100 d) ½ e) undefined.

14.

15. a) $(-\infty, \infty)$ b) $(-\infty, 0) \cup (0, \infty)$ c) $(-\infty, 1) \cup (1, \infty)$

16. a) 1 b) $-\infty$ c) DNE

17. a) ½ b) -1/4

18. -6 19. a) 3/2 b) 0.5