MAC 2311
Test 1 Review

Text book pp.115-116 # 3, 5, 7, 13-23 (odd), 29, 33, 37, 39, 43, 55, 57, 59, 71, 73, 77, 89.
p. 200 # 1, 5
p. 279 # 63, 67, 71, 75, 77

1. Find each of the following limits or state that the limit does not exist.

1. $\lim_{x\to 0}\frac{\sin 4x}{x}$ 2. $\lim_{x\to 1}e^{x^2-x}$ 3. $\lim_{x\to 0}\frac{1-\cos x}{x\cos x}$

4. $\lim_{x\to 3}\left[4x^3+3x^2-2x+1\right]$ 5. $\lim_{x\to 3}\frac{2x-3}{x+5}$ 6. $\lim_{x\to -5}\frac{2x-3}{x+5}$

7. $\lim_{x\to 4} g\left(f\left(x\right)\right)$, if $f\left(x\right)=2x^2-3x+1,\ g\left(x\right)=\sqrt[3]{x+6}$

8. $\lim_{x\to 2}\left(\sqrt{x-3}\right)$ 9. $\lim_{t\to 0}\frac{\sqrt{t+2}\sin t}{t}$ 10. $\lim_{t\to 0}\frac{\cos t-\cos^2(t)}{t}$

11. $\lim_{x\to 4}\frac{4-x}{\sqrt{x}-2}$ 12. $\lim_{x\to \pi}\frac{\pi x-\pi^2}{2x-2\pi}$ 13. $\lim_{x\to 0}\frac{\tan(x)}{x}$

14. $\lim_{x\to 3}\frac{(9-x^2)}{x-3}$ 15. $\lim_{x\to -\infty}\frac{-1}{2x}$ 16. $\lim_{x\to \infty}\frac{x-2}{\sqrt{x^2+7}}$

17. $\lim_{x\to 0^+}\frac{1}{x^2}$ 18. $\lim_{x\to -4^-}\frac{x}{x+4}$ 19. $\lim_{x\to 0}\frac{\sin(x)\cos(x)}{x}$

20. $\lim_{x\to \infty}\frac{1-x}{3x^2-7x+4}$ 21. $\lim_{x\to -3}\frac{|x+3|}{x+3}$ 22. $\lim_{x\to 1}\left(x^3+\sqrt{x^2-1}+x\right)$

23. $\lim_{x\to k}\frac{x^2-k^2}{k^4-x^4}$ 24. $\lim_{t\to 0}\frac{(t^2-3)^2-9}{t^2}$ 25. $\lim_{x\to 4}\sin^{-1}(\frac{x}{4})$

26. $\lim_{x\to \infty}2+e^{-x}$ 27. $\lim_{x\to 4}\frac{x^2-1}{x^2+1}$ 28. $\lim_{x\to 0}\frac{\sin\left(x^2\right)}{2x^2}$

29. $\lim_{x\to \infty}\frac{6-x^3}{7x^3+3}$ 30. $\lim_{x\to -\infty}2+e^{-x}$ 31. $\lim_{x\to -2}\frac{2x}{x^2+3x+2}$

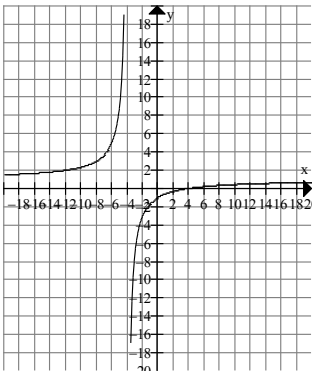
32. $\lim_{x\to 4^-}\frac{|x-4|}{x-4}=$ 33. $\lim_{x\to 2^+} f(x)$ if $f(x)=\begin{cases}3x-5, x\le 2\\ 0.5x^2, x>2\end{cases}$

34. $\lim_{x\to 0}\frac{x}{\sin(5x)}$ 35. $\lim_{x\to 0}x^3\sin\left(\frac{1}{\sqrt[3]{x}}\right)$ 36. $\lim_{x\to -2^+}[\![x+5]\!]$

2. Are the following functions continuous on the given intervals? Explain, why or why not?

a) $\dfrac{1}{\cos(x)}$ *on* $[0,\pi]$ b) $\dfrac{e^x}{e^x-1}$ *on* $[-1,\ 1]$ c) $\dfrac{1}{x-2}$ *on* $[-1,1]$

3. Evaluate the limits from the following graph of g(x).



a) lim g(x)
x→+∞

b) lim g(x)
x→-4^+

c) lim g(x)
x→-∞

d) lim g(x)
x→-4^-

e) lim g(x)
x→-4

4. Give numerical evidence to conjecture the value of $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x$.

x	1000	10000	1000000	10000000
f(x)				

5. Using a table, evaluate

$\lim\limits_{x-->2}(x^2-2x)$

x	*1.923*	*1.95*	*1.999*	*2*	*2.001*	*2.10*	*2.15*
f(x)							

6. Fill in the table and guess the value of the limit.

$$\lim_{x\to 2} f(x),\ where\ f(x)=\frac{x^2-x-2}{x^2+x-6}.$$

x	f(x)		x	f(x)
2.002			1.998	
2.001			1.999	
2.0001			1.9999	

7. Graph the function and use the graph to estimate the value of the limit.

a) $\lim_{\theta \to 0} \frac{\sin(3\theta)}{\sin(2\theta)}$ b) $\lim_{x \to 0} \frac{2^x - 1}{4^x - 1}$

8. Let $\lim_{x \to -1} f(x) = 2$, $\lim_{x \to -1} g(x) = -3$, *and* $\lim_{x \to -1} h(x) = 4$ and find each limit:

a) $\lim_{x \to -1} \left[2f(x) - 3h(x) \right]$ b) $\lim_{x \to -1} \sqrt{f(x)h(x)}$

9. Which of the functions is continuous at $x = 5$? Explain.

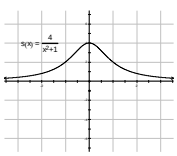
a) $f(x) = \frac{1}{x}$ b) $f(x) = \frac{1}{5 - x}$ c) $f(x) = \begin{cases} \frac{x^2 - 25}{x - 5}, & x \neq 5 \\ 10, & x = 5 \end{cases}$

10. Determine if the function is continuous at the given value of a.

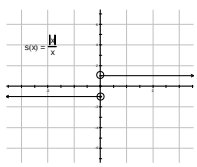
$$f(x) = \begin{cases} x^2, & x > 1 \\ -x + 2, & x \leq 1 \end{cases} \qquad a = 1$$

11. Determine the intervals where the following functions are continuous.

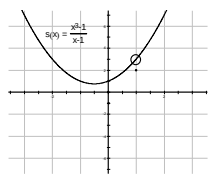
a)



b)



c)



12. Evaluate the limit assuming that $\lim_{x \to -4} f(x) = 3$ and $\lim_{x \to -4} g(x) = 1$.

a) $\lim_{x \to -4} f(x)g(x)$ b) $\lim_{x \to -4} (2f(x) + 3g(x))$

c) $\lim_{x \to -4} \frac{g(x)}{x^2}$ d) $\lim_{x \to -4} \frac{f(x) + 1}{3g(x) - 9}$

13. Determine the value of a so that the function is continuous.

$$f(x) = \begin{cases} ax + 4, & x < 1 \\ x^2 - 2a, & x \geq 1 \end{cases}$$

14. Use that the intermediate value theorem to show that $f(x) = x^4 - 2x^2 + 3x$ has a zero in the interval $[-2, -1]$. Do not find the zero.

15. Find the horizontal asymptote of each function if any.

a) $\frac{2x^2}{4 - x^2}$ b) $\frac{x}{x^3 + 6x^2 + x}$ c) $\frac{1}{2 + e^{-t}}$

d) $\dfrac{x^4-1}{x^2+1}$

16. Determine all vertical asymptotes of $f(x)=\dfrac{x+2}{x^2-2x-8}$.

17. Use the definition of a limit, for a given $\varepsilon = 0.01$, find $\delta > 0$

a) $\lim\limits_{x\to 1}(4x-1)=3$

b) $\lim\limits_{x\to -2}(3x+5)=-1$

18. Use the definition of a derivative only (limit process) to calculate the derivative of

a) $f(x)=x^2-3x+1$ b) $y=\dfrac{1}{x-1}$

19. Use the alternative definition of a derivative to find $f'(x)$ at the given x-value.

$f(x)=x^2+x$ when x = 5

20. Let $f(x)=\sqrt{x}$

a. Use the **definition** of the derivative to calculate the derivative of f.

b. Use (a) to find the **slope** of the tangent line to the graph of $f(x)$ at $x = 9$.

c. Find the equation of the tangent line at (9, 3).

Circle the best answer:

21. Find the limit: $\lim\limits_{x\to 0}\dfrac{\frac{1}{x+3}-\frac{1}{3}}{x}$

a) 0 b) $\frac{1}{9}$ c) $-\frac{1}{9}$ d) Limit does not exist

e) None of these

22. The function $f(x)=\sqrt{(16-x^2)}$ is continuous on

a) $(-4,4)$ b) $(-\infty,\infty)$ c) $[-4,4]$

d) $[0,4]$ e) None of these

23. $\lim_{\Delta x \to 0} \frac{3(x+\Delta x)^2 - 3x^2}{\Delta x}$

a) 0 b) $2x$ c) $6x + \Delta x$ d) Limit does not exist

e) None of these

24. Find the limit: $\lim_{x \to 1} f(x)$ if $f(x) = \begin{cases} -2x^2 - 1, x \neq 1 \\ 3, \quad x = 1 \end{cases}$

a) 3 b) -3 c) 1 d) Limit does not exist

e) None of these

25. If $\lim_{x \to c} f(x) = \frac{1}{2}$ and $\lim_{x \to c} g(x) = -\frac{1}{3}$, then $\lim_{x \to c} (-3f(x) - g(x)) =$

a) 1/2 b) $-1/2$ c) $-5/2$ d) $5/2$

e) None of these

26. For what value of k is the function f continuous if $f(x) = \begin{cases} kx - 3 & if \quad x \leq 3 \\ kx^2 + 5 & if \quad x > 3 \end{cases}$

a) -4/3 b) 0 c) 14 d) 4/3
e) None of these

27. Find Horizontal asymptote(s) for the graph of the following function.

$g(x) = \frac{3x^2 - 6}{\sqrt{x^4 + 2}}$

a) y = 3 b) No Horizontal Asymptote c) y = 3, y = -3

d) y = 0 e) None of these

Answers:
1.

1.	4	2.	1	3.	0	4.	130	5.	3/8
6.	DNE	7.	3			8.	DNE	9.	$\sqrt{2}$
10.	0	11.	-4			12.	$\frac{\pi}{2}$	13.	1

14. -6	15.	0	16.	1	17.	∞	
18. ∞	19.	1	20.	0	21.	DNE	
22. 2	23.	$\frac{-1}{2k^2}$	24.	-6	25.	$\frac{\pi}{2}$	
26. 2	27.	15/17	28.	½	29.	-1/7	
30. ∞	31.	DNE	32.	-1	33.	2	
34. 1/5	35.	0	36.	3			

2.a) No b) No c) Yes

3. a) 1 b) $-\infty$ c) 1 d) ∞ e) DNE

4. i) e = 2.7183 5. 0 6. .6

7.a) 3/2 b) 0.5 8. a) -8 b) $2\sqrt{2}$

9.a, c 10. Yes

11. $(-\infty, \infty)$ b) $(-\infty, 0) \cup (0, \infty)$ c) $(-\infty, 1) \cup (1, \infty)$

12. a) 3 b) 9 c) 1/16 d) -2/3

13. -1 15. a) -2 b) 0 c) ½, 0

d) undefined. 16. x = 4 17. a) .0025 b) .003

18. a) $2x-3$ b) $\frac{-1}{(x-1)^2}$ 19. 11

20. a) $\frac{1}{2\sqrt{x}}$ b) 1/6 c) $y = \frac{1}{6}x + \frac{3}{2}$ 21. 22. C

23. e 24. b 25. e 26. a 27. a