MAC 2311 – FINAL EXAM – REVIEW

1. Find each of the following limits if it exists,

a) $\lim_{x\to 2}(3x^2+5)$. (17) b) $\lim_{x\to -1}\frac{x^2+3x+2}{x^2+1}$. (0)

c) $\lim_{x\to 3}\sqrt{x^2-4}$. **(** $\sqrt{5}$ **)** d) $\lim_{x\to 2}\frac{x-2}{x^2-4}$. (1/4)

e) $\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$ (1/4) f) $\lim_{x\to 1}\frac{x^2+x-2}{x^2-3x+2}$ (-3)

g) $\lim_{x\to\infty}\frac{\sin x}{x^2}$ (0) h) $\lim_{x\to 5}\frac{x-5}{|x-5|}$. DNE

i) $\lim_{x\to\infty}\frac{2x^2+x-2}{3x^2-3x+2}$ (2/3) j) $\lim_{x\to -\infty}\frac{\sqrt{4x+x^2}}{4x+1}$ (-1/4)

k) $\lim_{x\to 1}\frac{x^2+7x-8}{x-1}$ (9) l) $\lim_{x\to 0}\frac{\sin^2(3x)}{x^2}$ (9)

m) $\lim_{x\to -1^-}\frac{x^3}{(x+1)^2}$ ∞ n) $\lim_{x\to 3^+}\frac{x}{x-3}$ **(** ∞ **)**

o) $\lim_{x\to 4}\frac{x^2+2x-7}{x-4}$ DNE p) $\lim_{x\to\frac{\pi}{4}}\frac{\sin(x-\frac{\pi}{4})}{x-\frac{\pi}{4}}$ (1)

q) $\lim_{x\to 2^-}\frac{1}{x-2}$. **(** $-\infty$ **)** r) Find $\lim_{x\to 2}\frac{1}{(x-2)^2}$. **(** ∞ **)**

s) $\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}$. (-1/9) t) Find $\lim_{x\to\infty}\frac{4x^2-8x}{8x^2+6x+5}$. (1/2)

u) $\lim_{t\to 0}\frac{\sin 3t}{\sin 5t}$ (3/5) v) $\lim_{x\to 0}\frac{\sin 3x}{\frac{x}{3}}$ (9)

w) $\lim_{x\to 3}f(x)$ Where $f(x)=\begin{cases}x^2 & \text{if } x\le 3\\ 9-x & \text{if } x>3\end{cases}$ DNE

2. If $\lim_{x\to c}f(x)=-\frac{1}{2}$ and $\lim_{x\to c}g(x)=\frac{2}{3}$, find $\lim_{x\to c}\frac{f(x)}{g(x)}$. (-3/4)

3. Use definition of the limit to show that $\lim_{x\to 2}(2x-1)=3$, find δ. ($\delta=\frac{\varepsilon}{2}$)

4. A function f is defined by $f(x)=\begin{cases}\dfrac{x^2-9}{x-3} & \text{if } x\neq 3\\ k & \text{if } x=3\end{cases}$, if f is continuous at $x=3$, what is the value of k? (6)

5. If a ball is thrown into the air with a velocity of 40 ft. /s, its height (in feet) after t seconds is given by $s(t) = 40t - 16t^2$. Find the velocity when t = 2. (–24 ft. /s)

6. Find an equation of the tangent line to the curve at the given point
$y = 2xe^x$, (0,0) (y = 2x)

7. Use a linear approximation $L(x)$ to an appropriate function $f(x)$, with an appropriate value of a, to estimate $\sqrt[3]{25}$. ($\sqrt[3]{25} \approx \frac{79}{27} \approx 2.9259$)

8. Differentiate:

a) $f(x) = e^{-5x}\cos 3x$ $f'(x) = -e^{-5x}(3\sin 3x + 5\cos 3x)$

b) $y = (x^2)\cdot f(x)$ $\frac{dy}{dx} = x^2 f'(x) + 2x\, f(x)$

c) $f(x) = \frac{3x}{x^2-1}$ $\frac{3(-1-x^2)}{(x^2-1)^2}$

d) $y = \tan^{-1}(e^x)$ $y' = \frac{e^x}{1+e^{2x}}$

e) $x = \cos y$ -csc(y)

f) $y = \frac{2x}{1-3x^2}$ $\frac{-2}{1-3x^2}$

g) $y = x^3\sqrt{2x+1}$ $\frac{x^2(7x+3)}{\sqrt{2x+1}}$

h $y = \cot^3\sqrt{x}$ $\frac{-3}{2\sqrt{x}}\cot^2\sqrt{x}\csc^2\sqrt{x}$

i) $y = x^x$ $y' = x^x(\ln x + 1)$

j) $y = \sin^2 x - \cos^2 x$ 2 sin (2x)

k) $y = \ln(\cos x)$ $-\tan(x)$

l) $f(x) = sin^3 x$ $3\sin^2(x)\cos(x)$

m) $y = 5\sin^4(x^3 - 3x^2)$. $(60x^2 - 120x)\cos(x^3-3x^2)\sin^3(x^3-3x^2)$

9. a) Find all vertical asymptotes of $f(x)=\dfrac{x-3}{x^2+x-6}$. (-3, 2)

b) Find all horizontal asymptotes of $g(x)=\dfrac{2x^2+3}{3x^2+x-6}$. (2/3)

10. Determine the value of c for which $f(x)$ is continuous for all real numbers when
$f(x)=\begin{cases} x-2, & x\le 5 \\ cx-3, & x>5 \end{cases}$. (6/5)

11. Find y'', if $y=\dfrac{x^6}{6}-\dfrac{x^5}{5}+x^2-2$ $5x^4-4x^3+2$

12. Consider the relation $x^2+y^2=4$. Find the slope of the tangent line at the point (2, -1)
(2)

13. Determine the value of c that satisfies the Mean Value Theorem for derivatives for $y=x^3$ on the interval [0, 3]. ($c=\sqrt{3}$)

14. Calculate $\dfrac{d^2y}{dx^2}$ for $y=\dfrac{1-x}{2-x}$. $\dfrac{-2}{(2-x)^3}$

15. Find an equation for the tangent line to the graph of $f(x)=\sqrt{x+1}$ at the point where x = 3.
4 y – x = 5

16. The position function for the movement of a particle is given by $s=(t^3+1)^2$ where s is measured in feet and t is measured in seconds. Find the acceleration of the particle at 1 second. (42)

17. Find the values of x for all points on the graph of $f(x)=x^3-2x^2+5x-16$ at which the slope of the tangent line is 4. (1/3, 1)

18. As a balloon in the shape of a sphere is being blown up, the volume increases at a rate of 4 cubic inches per second. At what rate is the radius increasing when the radius is 1 inch?
$v=(4/3)\pi r^3$ $(\dfrac{1}{\pi})$

19. An object projected vertically from a height of 80 feet with an initial velocity of 64 ft. / sec has a height, h (t), given by $h(t)=-16t^2+64t+80$ where t is time in seconds. Find the velocity at which the object hits the ground. (-96 ft. / sec)

20. Find the differential dy.

a) If $y=\left(x^2-2\right)^3$. $dy=6x\left(x^2-2\right)^2dx$

b) If y = sec (3x). dy = $3\sec(3x)\tan(3x)dx$

c) $y=(x^2+5)^3,\quad x=1,\quad dx=0.05$ $dy=6x(x^2+5)^2dx$ =10.8

21. Determine the value of c that satisfies the Mean Value Theorem for derivatives for

$f(x) = x^3$ on $[0,1]$. $\frac{1}{\sqrt{3}}$

22. Determine intervals where $f(x) = \frac{1}{x^2}$ is increasing or decreasing. Dec (-∞, 0) U (0, ∞)

23. Find all critical numbers for the function $f(x) = x\sqrt{2x+1}$. -1/3, -1/2

24. Find all extrema in the interval [0, 2π] if $y = \sin x + \cos x$. $(\frac{\pi}{4}, \sqrt{2}), (\frac{5\pi}{4}, -\sqrt{2})$

25. Let R be the region bounded by $y = x^2 - 4x + 6$, $x = 1$, $x = 4$, and $y = 0$. Find the area of the region. (9)

26. Find the area determined by the two curves $f(x) = x$ and $g(x) = x^2$ (1/6)

27. A rectangular box is to be made by cutting out equal squares from each corner of a piece of cardboard 10 inches by 16 inches and then folding up the sides. What must be the length of the side of the square cut out if the volume is to be maximized? What is the maximum volume?

square cut-out: 2 inches by 2 inches; volume: 144 in^3

28. If $\int_0^1 f(x)\,dx = 2$, $\int_1^2 f(x)\,dx = 3$, $\int_0^1 g(x)\,dx = -1$ and $\int_0^2 g(x)\,dx = 4$, use the properties of definite integrals to calculate each of the following integrals.

(a) $\int_0^2 [2f(x) + g(x)]\,dx$ 14

(b) $\int_0^1 [2f(x) + g(x)]\,dx$ 3

(c) $\int_2^1 [2f(x) + 5g(x)]\,dx$ -31

(d) $\int_1^1 [3f(x) + 2g(x)]\,dx$ 0

(e) $\int_0^2 [\sqrt{3}f(x) + \sqrt{2}g(x) + \pi]\,dx$ $5\sqrt{3} + 4\sqrt{2} + 2\pi$

29. Find all intervals for which the graph of $y = 8x^3 - 2x^4$ is concave downward.

(-∞, 0) U **(2, ∞)**

30. Let $f(x) = x^3 - x^2 + 3$. Use the second derivative test to find all relative extrema.

Max @ x=0, Min @ x = 2/3

31. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up. $(-\infty, -2) \cup (2, \infty)$

32. Determine whether Rolle's Theorem can be applied to the function, $f(x) = (x-2)(x+3)^2$ on the interval [–3, 2]. If it can, then find all values of c guaranteed by the theorem.

-3, 1/3

33. The sum of two numbers is 24. Find the maximum of the product of the two numbers.

(144)

34. Let $f(x) = \frac{3x+6}{x-1}$.

(a) Find the intervals on which the graph is increasing and decreasing.
(b) Find any local extrema.
(c) Find the intervals on which the graph is concave up and concave down.
(d) Find any inflection points.
(e) Find all asymptotes.
(a) Dec: $(-\infty, 1)$, $(1, \infty)$
(b) No local extrema
(c) Concave upward: $(1, \infty)$; concave downward: $(-\infty, 1)$
(d) No inflection points
(e) Horizontal asymptote: $y = 3$; vertical asymptote: $x = 1$

35. Find all points of inflection for the graph of the function $f(x) = 2x(x-4)^3$. (2, -32), (4, 0)

36. The sum of two numbers is 12. Find the two numbers that would give a maximum product. 6, 6

37. Find the maximum value and minimum value of $f(x) = (x-3)^{2/3}$ on [0, 4].

Max: $f(0) = \sqrt[3]{9}$; min: $f(3) = 0$

38. Find the intervals for which the function $f(x) = x^4 - 4x^3$ is increasing or decreasing.

Dec $(-\infty, 0)$ U $(0, 3)$, Inc. $(3, \infty)$

39. If $y = \left(x^2+1\right)^4$ Find the differential dy. $8x(x^2+1)^3\,dx$

40. A spherical balloon is being blown up at a rate of 100 cm^3/min. At what rate is its radius r changing when r is 4 cm? $\frac{25}{16\pi} \approx 0.4974$ cm/min

41. Determine if the Mean Value Theorem (for derivatives) applies to $f(x) = \sin(3x)$ on the interval $\left[0, \frac{\pi}{4}\right]$. If it does, find the value of c whose existence is guaranteed by the Mean Value Theorem. (Use the inverse trigonometric functions on your calculator to approximate the value(s) for c.)

MVT applies since f (x) is continuous on $\left[0, \frac{\pi}{4}\right]$ and differentiable on $\left(0, \frac{\pi}{4}\right)$.

$C \approx 0.422$

42. Find an equation of the tangent line to the graph of the equation $x^2 + 9xy + y^2 = 36$ at the point (0, 6). $y = -\frac{9}{2}x + 6$

43. Find the average value function of $f(x) = \sin(x)$ over the interval $[0, \pi/2]$ $(2/\pi)$

44. Find the area determined by the 2 curves $f(x) = x$ and $g(x) = 2x^2$ 10/3

45. Evaluate each of the following integrals

a. $\int_1^9 \frac{1}{2x}\,dx$ ln 3

b. $\int_{\pi}^{2\pi} \cos\theta \, d\theta$ 0

C. $\int_1^2 \frac{3}{t^4}\, dt$ 7/8

d. $\int x(1+2x^4)dx$ $\frac{x^2}{2}+\frac{x^6}{3}+C$

e. $\int \left(2-\sqrt{x}\right)^2 dx$ $4x-\frac{8}{3}x^{3/2}+\frac{1}{2}x^2+C$

f. $\int (\sin x)^3 \cos x\, dx$ $\frac{\sin^4 x}{4}+C$

g. $\int_0^{\pi/6} \sin^3\theta \cos\theta\, d\theta$ $\frac{1}{64}$

h) $\int x^2\cos^5(x^3+5)\sin(x^3+5)\, dx$ $-\frac{1}{18}\cos^6(x^3+5)+C$

i) $\int \frac{6x-9}{x^2-3x+4}dx$ $3\ln(x^2-3x+4)+c$

j) $\int x\sqrt{x^2+4}\, dx$ $\frac{1}{3}(x^2+4)^{3/2}+C$

k) $\int \frac{x^4-x^3}{x^2}dx.$ $\frac{x^3}{3}-\frac{x^2}{2}+c$

l) $\int \sqrt[5]{x^2}dx.$ $\frac{5}{7}x^{7/5}$ +c

m) $\int \frac{\sec^3\theta\tan\theta}{1+\tan^2\theta}d\theta.$ $\sec(\theta)+c$

n) $\int_0^{\pi/2} \frac{\cos x}{1+\sin x}dx$? ln (2)

o) $\int x(x^2-1)^4 dx.$ $\frac{1}{10}(x^2-1)^5+c$

p) $\int \sin^3(3x)\cos(3x)dx.$ $\frac{\sin^3(3x)}{12}+c$

q) $\int 3\csc^2 x dx.$ $-3\cot(x)$

r) $\int_{-1}^{1}|x|dx.$ (1)

s) $\int x\sqrt{x+1}dx.$ $\frac{2}{15}(x+1)^{3/2}(3x-2)$

t) $\int_0^2 \frac{t^3}{\sqrt{t^4+9}}\, dt$ 1

u) $g(x) = \int_0^x \sqrt[4]{1+t^4}\, dt$ $g'(x) = \sqrt[4]{1+x^4}$

v) $\int \frac{x^2+1}{x^2} dx$ $x - \frac{1}{x} + C$

w) $g(x) = \int_{x^2-3x}^{0} (t + \sin t)\, dt$ $g'(x) = -(2x-3)[(x^2-3x) + \sin(x^2-3x)]$

x) $\int_{-\pi/2}^{\pi/2} (2x + \cos x)\, dx$ 2

46. Estimate the area under the graph of $f(x) = \frac{1}{x}$ from x = 1 to x = 5 using four rectangles and right endpoints. Repeat using left endpoints. $R_4 = 77/60 \quad L_4 = 25/12$

47. If a snowball melts so that its surface area decreases at a rate of 1 sq. cm/min, find the rate at which the diameter decreases when the diameter is 10 cm. (Note: $S = 4\pi r^2$)

Decreases at a rate of $\frac{1}{20\pi}\ cm/\min$

48. For each of the following functions: Find the intervals where it is increasing and decreasing. Find the local maximum and minimum values. Find the intervals of concavity and the inflection points.

a) $f(x) = \frac{x}{(x-1)^2}$ x = 1 vertical asymptote

(-1, -¼) local minimum
CU: (-2, 1) and (1, ∞) Inflection point (-2, -2/9

b) $f(x) = 3x^5 - 5x^3 + 3$ local maximum: (-1, 5) local minimum: (1, 1)

Inflection points: $\left(\pm\frac{1}{\sqrt{2}}, 3 \pm \frac{7\sqrt{2}}{8}\right)$

49. Find the average value of the given function on the indicated interval.

(a) $f(x) = 8 - x^2$; [0, 9] -19

(b) $g(t) = 8\sqrt{t}$; [1, 36] $\frac{688}{21}$

(c) $f(x) = 3x\sin(x^2)$; $[0, \sqrt{\pi}\,]$ $\frac{3}{\sqrt{\pi}}$

50. Find the area of the region enclosed by the curves $x = 1 - y^2, \quad x = y^2 - 1$ 8/3

51. If $f(x) = \cos[\cos(\pi x)]$, find f ′ (1.1). $f'(1.1) \approx 0.7903$