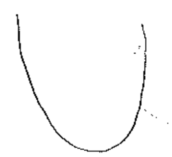
1w. $f(x)= \begin{cases} x^2 & \text{if } x \leq 3 \\ 9-x & \text{if } x > 3 \end{cases}$

$$\lim_{x \to 3} f(x) = DNE.$$

$$\lim_{x \to 3^-} (3)^2 = 9.$$



$$\lim_{x \to 3^+} 9-3 = 6$$

$$\lim_{x \to 3^+} f(x) \neq \lim_{x \to 3^-} f(x)$$

Thus the limit does not exist

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5. $s(t)= 40t-16t^2$

$v(t)=s'(t) = 40 - 32t$

$$v(2) = 40-32(2)$$
$$= 40-64$$
$$= -24$$

4. $f(x)=\begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x\neq 3 \\ K & \text{if } x=3 \end{cases}$

Our only concern is at $x=3$ since the domain of $\frac{x^2-9}{x-3}$ is all real #'s except for $x=3$.

What is K if f is continuous?

The conditions of continuity are...

① $\lim_{x\to 3} f(x)$ exist

② $f(3)$ is defined

③ $\lim_{x\to 3} f(x) = f(3)$

Examine each condition

1) $\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x+3)(x-3)}{(x-3)} = \lim_{x\to 3} x+3 = 3+3=6$

② $f(3)=K$

3) For f to be continuous

$$\lim_{x\to 3} f(x) = f(3)$$

$$6 = K$$

Thus $K=6$

2. $\displaystyle \lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \frac{-\frac{1}{2}}{\frac{2}{3}} = -\frac{1}{2} \div \frac{2}{3}$

$\displaystyle = -\frac{1}{2} \cdot \frac{3}{2} = -\frac{3}{4}$

8c. $\displaystyle f(x) = \frac{3x}{x^2-1}$

$\displaystyle f'(x) = \frac{(x^2-1)(3) - 3x(2x)}{(x^2-1)^2}$

$\displaystyle = \frac{3x^2 - 3 - 6x^2}{(x^2-1)^2}$

$\displaystyle = \frac{-3x^2-3}{(x^2-1)^2} = \frac{3(-x^2-1)}{(x^2-1)^2}$

8i. $\left\{ \begin{array}{l} y = x^x \\ \ln y = \ln x^x \\ \ln y = x \ln x \end{array} \right.$

$\displaystyle \frac{d}{dx}[\ln y] = \frac{d}{dx}[x \cdot \ln x]$

$\displaystyle \frac{y'}{y} = \cancel{x} \cdot \frac{1}{\cancel{x}} + \ln x \cdot 1$

$\displaystyle \frac{y'}{y} = 1 + \ln x$

$y' = y(1 + \ln x)$

$y' = x^x(1 + \ln x)$

8h. $y = \cot^3 \sqrt{x}$

$y = (\cot \sqrt{x})^3$

$y' = 3(\cot \sqrt{x})^2 \cdot \frac{d}{dx}\left[\cot x^{1/2}\right]$

$= 3\cot^2\sqrt{x}\left(-\csc^2 x^{1/2} \frac{d}{dx}\left[x^{1/2}\right]\right)$

$= -3\cot^2\sqrt{x}\ \csc^2 x^{1/2}\ \frac{1}{2} x^{-1/2}$

$= -\frac{3}{2} x^{-1/2} \cot^2\sqrt{x}\ \csc^2\sqrt{x}$

8j. $y = \sin^2 x - \cos^2 x$

$y' = 2\sin x \cos x - 2\cos x(-\sin x)$

$= 2\sin x \cos x + 2\cos x \sin x$

$= 4\sin x \cos x$

$= 2\sin(2x)$

Double Angle Identity

$\boxed{\sin(2\theta) = 2\sin\theta\cos\theta}$

8m $y = 5\sin^4(x^3-3x^2)$

$= 5\left[\sin(x^3-3x^2)\right]^4$

$y' = 5\cdot 4\left[\sin(x^3-3x^2)\right]^3 \cdot \frac{d}{dx}\left[\sin(x^3-3x^2)\right]$

$= 20\sin^3(x^3-3x^2) \cdot \cos(x^3-3x^2)\frac{d}{dx}\left[x^3-3x^2\right]$

$= 20\sin^3(x^3-3x^2)\cdot\cos(x^3-3x^2)(3x^2-6x)$

$= (60x^2-120x)\cos(x^3-3x^2)\sin^3(x^3-3x^2)$

18. $V = \frac{4}{3}\pi r^3$ (volume of a sphere)

$\frac{d}{dt}\left[V\right] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

↑ change in volume over time

↑ change in radius over time

Given:

Volume increases 4 cubic inches per second.

$\frac{dV}{dt} = 4 \text{ in}^3/\text{sec}$

when $r = 1\text{in}$

$4 = 4\pi(1)^2 \cdot \frac{dr}{dt}$

$\frac{4}{4\pi} = \frac{4\pi \cdot \frac{dr}{dt}}{4\pi} \longrightarrow \frac{dr}{dt} = \frac{1}{\pi} \text{ in/sec}$

47. $S = 4\pi r^2$ Surface area of a sphere

Since the given information and the information we are looking for involves diameter make the substitution

$$\frac{d}{2} = \frac{2r}{2}$$

$$\frac{d}{2} = r$$

Given:

Surface area decreases 1 sq. cm/min

$$\frac{dS}{dt} = -1 \frac{cm^2}{min}$$

$$d = 10$$

$$S = 4\pi \left(\frac{d}{2}\right)^2$$

$$S = 4\pi \left(\frac{d^2}{4}\right)$$

$$S = \pi d^2$$

$$\frac{d}{dt}[S] = \frac{d}{dt}[\pi d^2]$$

$$\underset{\substack{\uparrow \\ \text{rate of} \\ \text{change} \\ \text{surface} \\ \text{area}}}{\frac{dS}{dt}} = 2\pi d \underset{\substack{\uparrow \\ \text{rate of} \\ \text{change} \\ \text{diametr}}}{\frac{dd}{dt}}$$

$$-1 = 2\pi\, 10 \frac{dd}{dt}$$

$$\frac{-1}{20\pi} = \frac{20\pi \frac{dd}{dt}}{20\pi} \rightarrow \frac{dd}{dt} = \frac{-1}{20\pi} \frac{cm}{min}$$

49. Average value $= \frac{1}{b-a}\int_a^b f(x)\,dx$

a. $f(x) = 8 - x^2 \quad [0,9]$

Average value $\frac{1}{9-0}\int_0^9 (8-x^2)\,dx$

$= \frac{1}{9}\int (8-x^2)\,dy$

$= \frac{1}{9}\left(8x - \frac{x^3}{3}\right)\Big]_0^9$

$= \frac{1}{9}\left(8(9) - \frac{9^3}{3}\right) - \frac{1}{9}\left(8(0) - \frac{0^3}{3}\right)$

$= \frac{8(9)}{9} - \frac{\cancel{9}\cdot 9\cdot 9}{\cancel{9}\cdot 3} - 0$

$= 8 - \frac{81}{3}$

$= 8 - 27$

$= -19$

b. $g(x) = 8\sqrt{t} \quad [1,36]$

Average value $= \frac{1}{36-1}\int_1^{36} 8\sqrt{t}\,dt$

$= \frac{1}{35}\int_1^{36} 8t^{1/2}\,dt$

$= \frac{1}{35}\,\frac{8t^{3/2}}{3/2}\Big]_1^{36}$

$\frac{16}{105}\,t^{3/2}\Big]_1^{36}$

$= \frac{16}{105}(36)^{3/2} - \frac{16}{105}(1)^{3/2}$

$= \frac{16}{105}\left(\sqrt{36}\right)^3 - \frac{16}{105}\left(\sqrt{1}\right)^3$

$= \frac{16}{105}(6)^3 - \frac{16}{105}(1)^3$

$= \frac{16}{105}(216) - \frac{16}{105}$

$$= \frac{3456}{105} - \frac{16}{105}$$

$$= \frac{3440}{105}$$

$$= \frac{688}{21}$$

49. c. $f(x) = 3x \sin(x^2) \quad [0, \sqrt{\pi}]$

$$\text{Average value} = \frac{1}{\sqrt{\pi}-0} \int_0^{\sqrt{\pi}} 3x \sin(x^2)\, dx$$

$$= \frac{1}{\sqrt{\pi}} \int_0^{\sqrt{\pi}} 3x \sin(x^2)\, dx$$

Let $u = x^2$

$\frac{du}{2x} = \frac{2x\,dx}{2x}$

$\frac{du}{2x} = dx$

If $x=0$

$u = 0^2$

$u = 0$

If $x = \sqrt{\pi}$

$u = (\sqrt{\pi})^2$

$= \pi$

$$= \frac{1}{\sqrt{\pi}} \int_0^{\pi} 3x \sin u \cdot \frac{du}{2x}$$

$$= \frac{1}{\sqrt{\pi}} \int_0^{\pi} \frac{3}{2} \sin u\, du$$

$$= \frac{3}{2\sqrt{\pi}} \int_0^{\pi} \sin u\, du$$

$$= \frac{3}{2\sqrt{\pi}} -\cos u \Big]_0^{\pi}$$

$$= \frac{-3}{2\sqrt{\pi}} \left(\cos(\pi) - \cos(0)\right)$$

$$= \frac{-3}{2\sqrt{\pi}} \left(-1 - 1\right)$$

$$= \frac{-3}{2\sqrt{\pi}}(-2) = \frac{3}{\sqrt{\pi}}$$

16. $s = (t^3+1)^2$ (position)

$s' = 2(t^3+1)(3t^2)$ (velocity)

$s' = 6t^2(t^3+1)$

$= 6t^5 + 6t^2$

$s'' = 30t^4 + 12t$ (acceleration)

$s''(1) = 30(1)^4 + 12(1)$

$= 30 + 12$

$= 42$

13. Mean Value Theorem for derivatives

Slope of [illegible]

Slope of tangent

$f(x) = x^3$

$ARC = f'(c)$

$[0,3]$

$\frac{f(b)-f(a)}{b-a} = 3c^2$

$f'(x) = 3x^2$

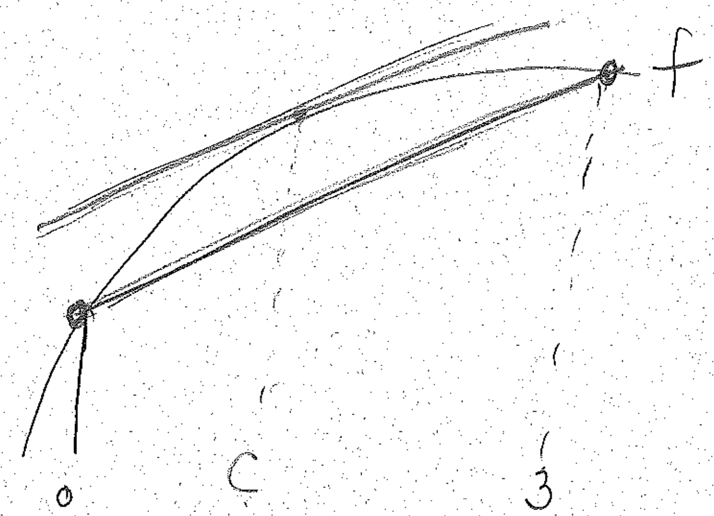
$f'(c) = 3c^2$

$\frac{f(3)-f(0)}{3-0}$

$\frac{3^3-0^3}{3}$

$\frac{27}{3}$

$= 9$



$\frac{9}{3} = \frac{3c^2}{3}$

$3 = c^2$

$\sqrt{3} = c$

18.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{d}{dt}\left[V\right] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cancel{3} r^2 \cdot \frac{dr}{dt}$$

rate of volume of sphrere

rat of radius

$$\frac{dV}{dt} = 4 \frac{in^3}{sec}$$

$$r = 1 \text{ in}$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$4 = 4\pi(1)^2 \cdot \frac{dr}{dt}$$

$$\frac{4}{4\pi} = \frac{4\pi \cdot \frac{dr}{dt}}{4\pi}$$

$$\frac{1}{\pi} = \frac{dr}{dt}$$