9.5 Practice Problems

Evaluate the expression

1. $\binom{7}{4}$

$= \frac{7!}{(7-4)!4!}$

$= \frac{7!}{3!4!} = \frac{7\cdot6\cdot5\cdot\cancel{4!}}{3!\cancel{4!}}$

$= \frac{7\cdot6\cdot5}{3\cdot2\cdot1} = 35$

2. $\binom{8}{1} = \frac{8!}{(8-1)!\cdot1!}$

$= \frac{8!}{7!\cdot1!} = \frac{8\cdot\cancel{7!}}{\cancel{7!}\cdot1}$

$= \frac{8}{1} = 8$

3. $\binom{9}{0} = \frac{9!}{(9-0)!0!}$

$= \frac{9!}{9!\cdot1!}$

$= 1$

4. $\binom{9}{3} = \frac{9!}{(9-3)!3!}$

$= \frac{9!}{6!\cdot3!}$

$= \frac{9\cdot8\cdot7\cdot\cancel{6!}}{\cancel{6!}\cdot3!}$

$= \frac{\overset{3}{\cancel{9}}\cdot\overset{4}{\cancel{8}}\cdot7}{\cancel{3}\cdot2\cdot1}$

$= 84$

Expand the expression using the binomial theorem.

5. $(x+2)^4$

$\frac{4\cdot3\cdot2\cdot1}{2!\cdot2!}$

$= \binom{4}{0}2^0x^4 + \binom{4}{1}2^1x^3 + \binom{4}{2}2^2x^2 + \binom{4}{3}2^3x + \binom{4}{4}2^4x^0$

$= 1\cdot1\cdot x^4 + 4\cdot2\cdot x^3 + 6\cdot4\cdot x^2 + 4\cdot8\cdot x + 1\cdot16\cdot1$

$= x^4 + 8x^3 + 24x^2 + 32x + 16$

6. $(x-4)^5$

$(x+(-4))^5$

$= \binom{5}{0}(-4)^0x^5 + \binom{5}{1}(-4)^1x^4 + \binom{5}{2}(-4)^2x^3 + \binom{5}{3}(-4)^3x^2 + \binom{5}{4}(-4)^4x + \binom{5}{5}(-4)^5x^0$

$= 1\cdot1\cdot x^5 + 5\cdot(-4)\cdot x^4 + 10\cdot16\cdot x^3 + 10(-64)x^2 + 5(256)x + 1\cdot1024\cdot1$

$= x^5 - 20x^4 + 160x^3 - 640x^2 + 1280x + 1024$

7. $(2x-3)^3$

$(2x+(-3))^3$

$= \binom{3}{0}(-3)^0(2x)^3 + \binom{3}{1}(-3)^1(2x)^2 + \binom{3}{2}(-3)^2(2x) + \binom{3}{3}(-3)^3(2x)^0$

$= 1\cdot1\cdot8x^3 + 3(-3)\cdot4x^2 + 3\cdot9\cdot2x + 1\cdot(-27)(1)$

$= 8x^3 - 36x^2 + 54x - 27$