

9.3 Practice Problems

Find the common ratio and write out the first 4 terms of the given geometric sequence.

$$1. \{a_n\} = 3^n$$

$a_1 = 3^1 = 3$ $a_2 = 3^2 = 9$ $a_3 = 3^3 = 27$ $a_4 = 3^4 = 81$	$\frac{9}{3} = 3$ $\frac{27}{9} = 3$ $\frac{81}{27} = 3$	common ratio $r = 3$
--	--	----------------------------

$$2. \{s_n\} = -3 \left(\frac{1}{3}\right)^n$$

common
ratio: $r = \frac{1}{3}$

$$S_1 = -3 \left(\frac{1}{3}\right)^1 = -3 \cdot \frac{1}{3} = -1$$

$$S_2 = -3 \left(\frac{1}{3}\right)^2 = -3 \cdot \frac{1}{9} = -\frac{1}{3}$$

$$S_3 = -3 \left(\frac{1}{3}\right)^3 = -3 \cdot \frac{1}{27} = -\frac{1}{9}$$

$$S_4 = -3 \left(\frac{1}{3}\right)^4 = -3 \cdot \frac{1}{81} = -\frac{1}{27}$$

$$4. \{s_n\} = 7^{n/5}$$

$$S_1 = 7^{1/5}$$

$$S_2 = 7^{2/5}$$

$$S_3 = 7^{3/5}$$

$$S_4 = 7^{4/5}$$

$$r = 7^{1/5}$$

$$3. \{e_n\} = \frac{3^{n-1}}{4}$$

$e_1 = \frac{3^{1-1}}{4} = \frac{3^0}{4} = \frac{1}{4}$ $e_2 = \frac{3^{2-1}}{4} = \frac{3^1}{4} = \frac{3}{4}$ $e_3 = \frac{3^{3-1}}{4} = \frac{3^2}{4} = \frac{9}{4}$	$e_4 = \frac{3^{4-1}}{4} = \frac{3^3}{4} = \frac{27}{4}$	common ratio $r = 3$
---	--	----------------------------

Find the fifth term and the nth term of the geometric sequence whose initial term a and common ratio r are given.

$$5. a=5; r=4$$

$$a_n = 5 \cdot 4^{n-1}$$

$$a_5 = 5 \cdot 4^{5-1}$$

$$= 5 \cdot 4^4$$

$$= 5 \cdot 256$$

$$= 1280$$

$$6. a=-7; r=2$$

$$a_n = -7(2)^{n-1}$$

$$a_5 = -7(2)^{5-1}$$

$$= -7(2)^4$$

$$= -7(16)$$

$$= -112$$

Find the indicated term of the geometric sequence.

$$7. 5^{\text{th}} \text{ term of } 3, \frac{3}{4}, \frac{3}{16}, \frac{3}{64}, \dots$$

$$\begin{array}{ccc} 3 & & \\ \uparrow & & \\ a_1 & r = \frac{3}{64} = \frac{3}{64} \cdot \frac{16}{3} & \\ & & = \frac{1}{4} \end{array}$$

$$a_n = 3 \left(\frac{1}{4}\right)^{n-1}$$

$$8. 6^{\text{th}} \text{ term of } 5, -10, 20, \underbrace{-40}_{\uparrow}$$

$$r = \frac{-40}{20} = -2$$

$$a_n = 5(-2)^{n-1}$$

$$\begin{aligned} a_6 &= 5(-2)^{6-1} \\ &= 5(-2)^5 \\ &= 5(-32) \\ &= -160 \end{aligned}$$

$$a_5 = 3 \left(\frac{1}{4}\right)^{5-1}$$

$$= 3 \left(\frac{1}{4}\right)^4$$

$$= \frac{3}{256}$$

Write a formula for the nth term of the geometric sequence.

9. $6, 36, 216, \dots$

$$\begin{array}{l} \uparrow \\ a_1 = 6 \\ r = \frac{216}{36} = 6 \end{array}$$

$$a_n = 6 \cdot 6^{n-1}$$

10. $-3, \frac{3}{5}, -\frac{3}{25}, \frac{3}{125}, \dots$

$$\begin{array}{l} \uparrow \\ -3 \\ r = \frac{\frac{3}{5}}{-3} = \frac{1}{5} \end{array}$$

$$r = \frac{3}{125} \cdot -\frac{25}{3} = -\frac{1}{5}$$

$$a_n = -3 \cdot \left(-\frac{1}{5}\right)^{n-1}$$

Find the nth term of the geometric sequence described below, where r is the common ratio.

11. $a_5 = 32, r = 2$

$$a_n = a_1 \cdot 2^{n-1}$$

$$32 = a_5 = a_1 \cdot 2^{5-1}$$

$$32 = a_1 \cdot 2^4$$

$$32 = 16a_1$$

$$2 = a_1$$

$$a_n = 2 \cdot 2^{n-1}$$

$$a_n = 2 \cdot 2^{n-1}$$

Find the sum.

12. $\frac{1}{5} + \frac{3}{5} + \frac{3^2}{5} + \frac{3^3}{5} + \dots + \frac{3^{n-1}}{5}$

$$\begin{array}{l} \uparrow \\ a_1 = \frac{1}{5} \\ r = \frac{\frac{3}{5}}{\frac{1}{5}} = \frac{3}{5} \end{array}$$

$$a_n = \frac{1}{5} \cdot 3^{n-1}$$

$$a_1 \cdot \frac{1-r^n}{1-r}$$

$$\begin{aligned} \sum_{k=1}^n \frac{1}{5} \cdot 3^{k-1} &= \frac{1}{5} \cdot \frac{1-3^n}{1-3} \\ &= \frac{1}{5} \cdot \frac{1-3^n}{-2} \\ &= -\frac{1}{10} (1-3^n) \end{aligned}$$

13. $\sum_{k=1}^n \left(\frac{2}{3}\right)^k = \sum_{k=1}^n \frac{2}{3} \left(\frac{2}{3}\right)^{k-1} = \frac{2}{3} \cdot \frac{1-\left(\frac{2}{3}\right)^n}{1-\frac{2}{3}}$

$$a_n = \left(\frac{2}{3}\right)^n$$

$$a_1 = \left(\frac{2}{3}\right)^1 = \frac{2}{3}$$

$$a_2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$a_3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

Re-write in
correct form

$$= \frac{2}{3} \cdot \frac{1-\left(\frac{2}{3}\right)^n}{\frac{1}{3}}$$

$$= \frac{3}{1} \cdot \frac{2}{3} \cdot \left(1 - \left(\frac{2}{3}\right)^n\right)$$

$$= 2 \cdot \left(1 - \left(\frac{2}{3}\right)^n\right)$$