Show that the sequence is arithmetic. Find the common difference and write out the first four terms.

1. $\{s_n\}=\{n+5\}$

$S_1 = 1+5=6$
$S_2 = 2+5=7$
$S_3 = 3+5=8$
$S_4 = 4+5=9$

$\boxed{d=1}$

2. $\{s_n\}=\{8-5n\}$

$S_1 = 8-5(1)=8-5=3$
$S_2 = 8-5(2)=8-10=-2$
$S_3 = 8-5(3)=8-15=-7$
$S_4 = 8-5(4)=8-20=-12$

$\boxed{d=-5}$

3. $\{s_n\}=\{4n+3\}$

$S_1 = 4(1)+3=7$
$S_2 = 4(2)+3=8+3=11$
$S_3 = 4(3)+3=12+3=15$
$S_4 = 4(4)+3=16+3=19$

$\boxed{d=4}$

4. $\{s_n\}=\{\frac{1}{3}n+\frac{1}{2}\}$ $\boxed{d=\frac{1}{3}}$

$S_1 = \frac{1}{3}(1)+\frac{1}{2} = \frac{1}{3}+\frac{1}{2} = \frac{2}{6}+\frac{3}{6}=\frac{5}{6}$
$S_2 = \frac{1}{3}(2)+\frac{1}{2} = \frac{2}{3}+\frac{1}{2} = \frac{4}{6}+\frac{3}{6}=\frac{7}{6}$
$S_3 = \frac{1}{3}(3)+\frac{1}{2} = 1+\frac{1}{2} = \frac{2}{2}+\frac{1}{2}=\frac{3}{2}$
$S_4 = \frac{1}{3}(4)+\frac{1}{2} = \frac{4}{3}+\frac{1}{2} = \frac{8}{6}+\frac{3}{6}=\frac{11}{6}$

Find the nth term of the arithmetic sequence $\{a_n\}$ whose initial term a_1 and common difference d are given. What is the fifty-first term?

5. $a_1=8; d=-4$

$a_n = 8+(n-1)(-4)$
$= 8-4n+4$
$= -4n+12$

$a_{51} = -4(51)+12$
$= -204+12$
$= -192$

6. $a_1=-3; d=5$

$a_n = -3+(n-1)5$
$= -3+5n-5$
$= 5n-8$

$a_{51} = 255-8$
$= 247$

Find the indicated term in the arithmetic sequence.

7. 100th term of $\underset{\underset{a_1}{\uparrow}}{4,7,}\underset{d=3}{\underbrace{10,13,}}16,...$

$a_n = a_1+(n-1)d$
$= 4+(n-1)3$
$= 4+3n-3$
$= 3n+1$

$a_{100} = 3(100)+1$
$= 300+1$
$= 301$

8. 57th term of $\underset{\underset{a_1}{\uparrow}}{-5,-3,}\underset{d=+2}{\underbrace{-1,1,}}3...$

$a_n = a_1+(n-1)d$
$= -5+(n-1)(2)$
$= -5+2n-2$
$= 2n-7$

$a_{57} = 2(57)-7$
$= 114-7$
$= 107$

$$\frac{n}{2}(a_1 + a_n) \qquad a_1 + (n-1)a$$

Find each sum.

9. $1, 6, 11, 16, 21, \dots, 5n-4$

$$\sum_{k=1}^{n} 5n-4 = \frac{n}{2}(1 + 5n - 4)$$
$$= \frac{n}{2}(5n-3)$$

10. $7, 13, 19, 25, 31, \dots, 301$

$$a_n = 7 + (n-1)6$$
$$a_n = 7 + 6n - 6$$
$$a_n = 6n + 1$$

$$\sum_{k=1}^{50} 6n+1 = \frac{50}{2}(7+301)$$
$$= 25(308)$$
$$= 7700$$

$$6n + 1 = 301$$
$$\frac{-1 \qquad -1}{6n = 300}$$
$$\frac{6n}{6} = \frac{300}{6}$$
$$n = 50$$

11. Find the first term and the common difference of the arithmetic sequence described. Give the recursive formula for the sequence. Find a formula for the nth term.

12th term is 4; 18th term is 28

a. What is the first term of the sequence? -40
b. What is the common difference? 4
c. What is the recursive formula?
d. What is the formula for the nth term of the sequence?

c) $a_1 = -40$
$a_n = a_{n-1} + 4$

$$a_{12} = a_1 + (12-1)d = 4$$
$$a_{18} = a_1 + (18-1)d = 28$$

$$a_1 + 11d = 4$$
$$a_1 + 17d = 28$$

$$\frac{-6d}{-6} = \frac{-24}{-6}$$
$$d = 4$$

$$a_1 + 11(4) = 4$$
$$a_1 + 44 = 4$$
$$a_1 = -40$$

d) $a_n = -40 + (n-1)4$
$$= -40 + 4n - 4$$
$$= 4n - 44$$

The Sartor Theatre has 25 seats in the first row and 50 rows in all. Each successive row contains one additional seat. How many seats are there in the theater?

$n = 50$

$$a_1 = 25$$
$$d = 1$$

$$a_n = a_1 + (n-1)d$$
$$= 25 + (n-1)1$$
$$= 25 + n - 1$$
$$= n + 24$$

$$\sum_{k=1}^{50} k + 24 = \frac{n}{2}(a_1 + a_n)$$
$$= \frac{50}{2}(25 + 74)$$
$$= 25(99)$$
$$= 2475$$