8.6 Practice Problems

Solve the following systems of nonlinear equations by substitution or elimination or graphically.

1. $\begin{cases} x-y=-1 \\ y=x^2+1 \end{cases}$

$x-(x^2+1)=-1$
$x-x^2-1=-1$
$0=x^2-x$
$0=x(x-1)$
$x=0 \quad x-1=0$
$x=1$
$(0,\ ) \quad (1,\ )$

$x=0$
$y=0^2+1$
$=1$
$(0,1)$

$x=1$
$y=1^2+1$
$=1+1$
$=2$
$(1,2)$

2. $\begin{cases} x^2+y^2=16 \\ y=x^2-4 \end{cases}$

$x^2+y^2=16$
$-x^2+4=-4$
$y^2+4=12$
$y^2+y-12=0$
$(y+4)(y-3)=0$
$y+4=0 \quad y-3=0$
$y=-4 \quad y=3$
$(\ ,-4) \quad (\ ,3)$

$y=-4$
$-4=x^2-4$
$+4 \quad +4$
$0=x^2$
$0=x$
$(0,-4)$

$y=3$
$3=x^2-4$
$+4 \quad +4$
$7=x^2$
$\pm\sqrt{7}=x$
$(\sqrt{7},3)$
$(-\sqrt{7},3)$

3. $\begin{cases} 3x^2+y^2=12 \\ x^2+y^2=4 \end{cases}$ ← mult by -1

new system
$3x^2+y^2=12$
$-x^2-y^2=-4$
$\frac{2x^2}{2}=\frac{8}{2}$
$x^2=4$
$x=\pm 2$
$(2,\ )(-2,\ )$

$x=2$
$2^2+y^2=4$
$4+y^2=4$
$y^2=0$
$y=0$
$(2,0)$

$x=-2$
$(-2)^2+y^2=4$
$4+y^2=4$
$y^2=0$
$y=0$
$(-2,0)$

4. $\begin{cases} xy=25 \\ y=x \end{cases}$

$x \cdot x=25$
$x^2=25$
$x=\pm 5$
$(5,\ )(-5,\ )$

$x=5$
$y=5$
$(5,5)$

$x=-5$
$y=-5$
$(-5,-5)$

5. $\begin{cases} 3x^2-2y^2=-5 \\ 2x^2-y^2=-2 \end{cases}$ ← mult by -2

new system
$3x^2-2y^2=-5$
$-4x^2+2y^2=4$
$-x^2=-1$
$x^2=1$
$x=\pm 1$
$(1,\ )(-1,\ )$

$x=1$
$2(1)^2-y^2=-2$
$2-y^2=-2$
$-2 \quad -2$
$-y^2=-4$
$y^2=4$
$y=\pm 2$
$(1,2)(1,-2)$

$x=-1$
$2(-1)^2-y^2=-2$
$2-y^2=-2$
$-2 \quad -2$
$-y^2=-4$
$y^2=4$
$y=\pm 2$
$(-1,2)(-1,-2)$

6. $\begin{cases} (x+3)^2+(y+4)^2=4 \\ y=x-3 \end{cases}$

$(x+3)^2+(x-3+4)^2=4$
$(x+3)^2+(x+1)^2=4$
$x^2+6x+9+x^2+2x+1=4$
$2x^2+8x+10=4$
$-4 \quad -4$
$2x^2+8x+6=0$
$x^2+4x+3=0$
$(x+3)(x+1)=0$
$x+3=0 \quad x+1=0$
$x=-3 \quad x=-1$
$(-3,\ ) \quad (-1,\ )$

$x=-3$
$y=-3-3$
$=-6$
$(-3,-6)$

$x=-1$
$y=-1-3$
$=-4$
$(-1,-4)$

7. $\begin{cases} y = x^2 - 6x + 5 \\ 2x + y = 1 \end{cases}$

$2x + x^2 - 6x + 5 = 1$

$x^2 - 4x + 5 = 1$

$-1 \quad -1$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x - 2 = 0 \quad x - 2 = 0$

$x = 2 \quad x = 2$

$(2, \ )$

$x = 2$

$2(2) + y = 1$

$4 + y = 1$

$-4 \quad -4$

$y = -3$

$(2, -3)$

8. $\begin{cases} 2x^2 + y^2 = 17 \\ x^2 + y^2 = 13 \leftarrow \text{mult by } -1 \end{cases}$

new system

$\begin{cases} 2x^2 + y^2 = 17 \\ -x^2 - y^2 = -13 \end{cases}$

$x^2 = 4$

$x = \pm 2$

$(2, \ ) \ (-2, \ )$

$x = 2$

$2^2 + y^2 = 13$

$4 + y^2 = 13$

$-4 \quad -4$

$y^2 = 9$

$y = \pm 3$

$(2, 3) \ (2, -3)$

$x = -2$

$(-2)^2 + y^2 =$

$4 + y^2 =$

$-4 \quad -$

$y^2 =$

$y = \pm$

$(-2, 3) \ (-2,$

9. $\begin{cases} y = x^2 \\ x^2 + y^2 = 6 \end{cases}$

$y + y^2 = 6$

$y^2 + y - 6 = 0$

$(y-2)(y+3) = 0$

$y - 2 = 0 \quad y + 3 = 0$

$y = 2 \quad y = -3$

$(\ , 2) \quad (\ , -3)$

$y = 2$

$2 = x^2$

$\pm\sqrt{2} = x$

$(\sqrt{2}, 2) \ (-\sqrt{2}, 2)$

$y = -3$

$-3 = x^2$

not possible

10. $\begin{cases} y = x \\ x^2 + y^2 = 8 \end{cases}$

$x^2 + x^2 = 8$

$\frac{2x^2}{2} = \frac{8}{2}$

$x^2 = 4$

$x = \pm 2$

$(2, \ ) \ (-2, \ )$

$x = 2$

$y = 2$

$(2, 2)$

$x = -2$

$y = -2$

$(-2, -2)$