8.5 Practice Problems

Write the improper fraction in proper form using long division.

1. $\frac{x^2+5}{x^2-4} = 1 + \frac{9}{x^2-4}$

$$\begin{array}{r} 1 \\ x^2-4 \overline{) x^2+5} \\ -x^2+4 \\ \hline 9 \end{array}$$

2. $\frac{3x^4+x^2-2}{x^3+8} = 3x + \frac{x^2-24x-2}{x^3-8}$

$$\begin{array}{r} 3x \\ x^3+8 \overline{) 3x^4+x^2-2} \\ -3x^4 \mp 24x \\ \hline x^2-24x-2 \end{array}$$

Write the partial fraction decomposition of each rational expression.

3. $\frac{5x+4}{x(x+2)}$

$$\frac{5x+4}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$$

$$x(x+2)\left(\frac{5x+4}{x(x+2)}\right) = x(x+2)\cdot\frac{A}{x} + x(x+2)\cdot\frac{B}{x+2}$$

$$5x+4 = A(x+2) + Bx$$

$$5x+4 = Ax+2A+Bx$$

$$\begin{cases} 5 = A+B \\ 4 = 2A \end{cases}$$

$$\frac{4}{2}=\frac{2A}{2}$$

$$2 = A$$

$$5 = 2+B$$

$$3 = B$$

$$\frac{5x+4}{x(x+2)} = \frac{2}{x} + \frac{3}{x+2}$$

4. $\frac{9x+1}{x^2+x-12}$

$$= \frac{9x+1}{(x+4)(x+3)} = \frac{A}{x+4} + \frac{B}{x-3}$$

$$9x+1 = A(x-3) + B(x+4)$$

$$9x+1 = Ax-3A+Bx+4B$$

$$\begin{cases} 9 = A+B \rightarrow A = 9-B \\ 1 = -3A+4B \end{cases}$$

$$1 = -3(9-B)+4B$$

$$1 = -27+3B+4B$$

$$1 = -27+7B$$

$$28 = 7B$$

$$4 = B$$

$$A = 9-4$$

$$= 5$$

$$\frac{9x+1}{x^2+x-12} = \frac{5}{x+4} + \frac{4}{x-3}$$

5. $\frac{5x^2+3x+1}{x^3+x^2} = \frac{5x^2+3x+1}{x^2(x+1)}$

$$\frac{5x^2+3x+1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$5x^2+3x+1 = Ax(x+1) + B(x+1) + Cx^2$$

$$5x^2+3x+1 = Ax^2+Ax+Bx+B+Cx^2$$

$$\begin{cases} 5 = A+C \\ 3 = A+B \\ 1 = B \end{cases}$$

$$A+1 = 3$$

$$A = 2$$

$$5 = 2+C$$

$$3 = C$$

$$\frac{5x^2+3x+1}{x^3+x^2} = \frac{2}{x} + \frac{3}{x^2} + \frac{3}{x+1}$$

6. $\dfrac{3x^2+1}{x^3+x} = \dfrac{3x^2+1}{x(x^2+1)}$

$$\begin{cases} 3 = A+B \\ 0 = C \\ A = 1 \end{cases} \qquad \begin{aligned} 3 &= 1+B \\ 2 &= B \end{aligned}$$

$$\frac{3x^2+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$\frac{3x^2+1}{x^3+x} = \frac{1}{x} + \frac{2x}{x^2+1}$$

$$3x^2+1 = A(x^2+1) + x(Bx+C)$$

$$0x + 3x^2 + 1 = Ax^2 + A + Bx^2 + Cx$$

7. $\dfrac{x^3+1}{(x^2+16)^2}$

$$\frac{x^3+1}{(x^2+16)^2} = \frac{Ax+B}{x^2+16} + \frac{Cx+D}{(x^2+16)^2}$$

$$\begin{cases} A = 1 \\ B = 0 \\ 16A + C = 0 \\ 16B + D = 1 \end{cases}$$

$$\begin{aligned} 16(1) + C &= 0 \\ 16 + C &= 0 \\ C &= -16 \end{aligned} \qquad \begin{aligned} 16(0) + D &= 1 \\ D &= 1 \end{aligned}$$

$$x^3+1 = (Ax+B)(x^2+16) + Cx + D$$

$$x^3+1 = Ax^3 + 16Ax + Bx^2 + 16B + Cx + D$$

$$\frac{x^3+1}{(x^2+16)^2} = \frac{x}{x^2+16} + \frac{-16x+1}{(x^2+16)^2}$$

Rewrite the improper fraction. Find the partial fraction decomposition of the proper fraction.

8. $\dfrac{x^5+x^4-x^2+2}{x^4-2x^2+1} = x+1+\dfrac{2x^3+x^2+x+1}{x^4-2x^2+1}$

$$\begin{array}{r} x+1 \\ x^4-2x^2+1 \overline{) x^5+x^4-x^2+2} \\ -x^5+2x^3 \mp x \\ \hline x^4+2x^3-x^2+x+2 \\ -x^4 \quad +2x^2 \quad \mp 1 \\ \hline 2x^3+x^2+x+1 \end{array}$$

$$\begin{cases} A + C = 2 \\ A + B - C + D = 1 \\ -A + 2B - C - 2D = 1 \\ -A + B + C + D = 1 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 1 & 1 & -1 & 1 & 1 \\ -1 & 2 & -1 & -2 & 1 \\ -1 & 1 & 1 & 1 & 1 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 5/4 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1/4 \end{array}\right]$$

$$= \boxed{x+1 + \frac{1}{x-1} + \frac{5/4}{(x-1)^2} + \frac{1}{(x+1)} + \frac{-1/4}{(x+1)^2}}$$

$$\frac{2x^3+x^2+x+1}{(x-1)^2(x+1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2}$$

$$2x^3+x^2+x+1 = A(x-1)(x+1)^2 + B(x+1)^2 + C(x-1)^2(x+1) + D(x-1)^2$$

$$2x^3+x^2+x+1 = A(x-1)(x^2+2x+1) + B(x^2+2x+1) + C(x^2-2x+1)(x+1) + D(x^2-2x+1)$$

$$2x^3+x^2+x+1 = A(x-1)(x^2+2x+1) + Bx^2+2Bx+B + C(x^3-2x^2+x+x^2-2x+1) + Dx^2-2Dx+D$$

$$2x^3+x^2+x+1 = A(x^3+2x^2+x-x^2-2x-1) + Bx^2+2Bx+B + C(x^3-x^2-x+1) + Dx^2-2Dx+D$$

$$2x^3+x^2+x+1 = A(x^3+x^2-x-1) + Bx^2+2Bx+B + Cx^3-Cx^2-Cx+C + Dx^2-2Dx+D$$

$$2x^3+x^2+x+1 = Ax^3+Ax^2-Ax-A+Bx^2+2Bx+B+Cx^3-Cx^2-Cx+C+Dx^2-2Dx+D$$

$$2x^3+x^2+x+1 = Ax^3+Cx^3+Ax^2+Bx^2-Cx^2+Dx^2-Ax+2Bx-Cx-2Dx-A+B+C+D$$