

8.4 Practice Problems

Use the following matrices to evaluate the given expression.

$$A = \begin{bmatrix} -2 & 3 \\ -5 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 9 & -3 \\ 2 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & 3 \\ -5 & 1 \\ 1 & 2 \end{bmatrix}$$

1. $A+B$

$$\begin{aligned} &= \begin{bmatrix} -2 & 3 \\ -5 & 1 \end{bmatrix} + \begin{bmatrix} 9 & -3 \\ 2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} -2+9 & 3+(-3) \\ -5+2 & 1+4 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ -3 & 5 \end{bmatrix} \end{aligned}$$

2. $3A$

$$\begin{aligned} &3 \begin{bmatrix} -2 & 3 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 3(-2) & 3(3) \\ 3(-5) & 3(1) \end{bmatrix} \\ &= \begin{bmatrix} -6 & 9 \\ -15 & 3 \end{bmatrix} \end{aligned}$$

3. $2A-4B$

$$2 \begin{bmatrix} -2 & 3 \\ -5 & 1 \end{bmatrix} - 4 \begin{bmatrix} 9 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -10 & 2 \end{bmatrix} - \begin{bmatrix} 36 & -12 \\ 8 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} -4-36 & 6+12 \\ -10-8 & 2-16 \end{bmatrix} = \begin{bmatrix} -40 & 18 \\ -18 & -14 \end{bmatrix}$$

4. AB

$$\begin{aligned} &\begin{bmatrix} -2 & 3 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 9 & -3 \\ 2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} -2(9)+3(2) & -2(-3)+3(4) \\ -5(9)+1(2) & -5(-3)+1(4) \end{bmatrix} \\ &= \begin{bmatrix} -18+6 & 6+12 \\ -45+2 & 15+4 \end{bmatrix} = \begin{bmatrix} -12 & 18 \\ -43 & 19 \end{bmatrix} \end{aligned}$$

5. BA

$$= \begin{bmatrix} 9 & -3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ -5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9(-2)+3(-5) & 9(3)+3(1) \\ 2(-2)+4(-5) & 2(3)+4(1) \end{bmatrix}$$

$$= \begin{bmatrix} -18+15 & 27+3 \\ -4-20 & 6+4 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 30 \\ -24 & 10 \end{bmatrix}$$

6. CA

$$\begin{bmatrix} -2 & 3 \\ -5 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ -5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2(-2)+3(-5) & -2(3)+3(1) \\ -5(-2)+1(-5) & -5(3)+1(1) \\ 1(-2)+2(-5) & 1(3)+2(1) \end{bmatrix}$$

$$= \begin{bmatrix} 4-15 & -6+3 \\ 10-5 & -15+1 \\ -2-10 & 3+2 \end{bmatrix} = \begin{bmatrix} -11 & -3 \\ 5 & -14 \\ -12 & 5 \end{bmatrix}$$

Find the inverse of each matrix.

7. $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array} \right] \xrightarrow{\substack{r_1 \rightarrow R_2 \\ r_2 \rightarrow R_1}} \left[\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 2 & 5 & 1 & 0 \end{array} \right]$$

$$\xrightarrow{r_1 + r_2 \rightarrow R_2} \left[\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 0 & -1 & 1 & -2 \end{array} \right] \xrightarrow{+1 \cdot r_2 \rightarrow R_2} \left[\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 0 & 1 & -1 & -2 \end{array} \right]$$

$$\xrightarrow{-3r_2 + r_1 \rightarrow R_1} \left[\begin{array}{cc|cc} 1 & 0 & 3 & -5 \\ 0 & 1 & -1 & -2 \end{array} \right]$$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & -2 \end{bmatrix}$$

Solve the system of equations by using the inverse matrices found above.

9. $\begin{cases} 2x + 5y = 8 \\ x + 3y = -1 \end{cases}$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 29 \\ -10 \end{bmatrix}$$

$$x = 29 \quad y = -10$$

Solution
(29, -10)

8. $\begin{bmatrix} 3 & -3 & 1 \\ -2 & 2 & -1 \\ -4 & 5 & -2 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 3 & -3 & 1 & 1 & 0 & 0 \\ -2 & 2 & -1 & 0 & 1 & 0 \\ -4 & 5 & -2 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\frac{1}{3}r_1 \rightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ -2 & 2 & -1 & 0 & 1 & 0 \\ -4 & 5 & -2 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\substack{2r_1 + r_2 \rightarrow R_2 \\ 4r_1 + r_3 \rightarrow R_3}} \left[\begin{array}{ccc|ccc} 1 & -1 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{2}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 1 & -\frac{2}{3} & \frac{4}{3} & 0 & 1 \end{array} \right] \xrightarrow{\substack{r_2 \rightarrow R_3 \\ r_3 \rightarrow R_2}} \left[\begin{array}{ccc|ccc} 1 & -1 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{2}{3} & \frac{4}{3} & 0 & 1 \\ 0 & 0 & -\frac{1}{3} & \frac{2}{3} & 1 & 0 \end{array} \right]$$

$$\xrightarrow{-3r_3 \rightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{2}{3} & \frac{4}{3} & 0 & 1 \\ 0 & 0 & 1 & -2 & -3 & 0 \end{array} \right] \xrightarrow{\substack{\frac{2}{3}r_3 + r_2 \rightarrow R_2 \\ \frac{1}{3}r_3 + r_1 \rightarrow R_1}} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -\frac{5}{3} & -1 & 0 \\ 0 & 1 & 0 & \frac{2}{3} & -2 & 1 \\ 0 & 0 & 1 & -2 & -3 & 0 \end{array} \right]$$

$$\xrightarrow{r_1 + r_2 \rightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & \frac{2}{3} & -2 & 1 \\ 0 & 0 & 1 & -2 & -3 & 0 \end{array} \right]$$

$$\begin{bmatrix} 3 & -3 & 1 \\ -2 & 2 & -1 \\ -4 & 5 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -2 & -3 & 0 \end{bmatrix}$$

10. $\begin{cases} 3x - 3y + z = -2 \\ -2x + 2y - z = 3 \\ -4x + 5y - 2z = 4 \end{cases}$

$$\begin{bmatrix} 3 & -3 & 1 \\ -2 & 2 & -1 \\ -4 & 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & -3 & 1 \\ -2 & 2 & -1 \\ -4 & 5 & -2 \end{bmatrix}^{-1} \begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -2 & -3 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ -5 \end{bmatrix}$$

$x = -1, y = -2, z = -5$
Solution
(-1, -2, -5)