

8.2 Practice Problems

1. Write the augmented matrix of the given system. $\begin{cases} 2x+3y=8 \\ 3x+4y=-5 \end{cases}$

$$\left[\begin{array}{cc|c} 2 & 3 & 8 \\ 3 & 4 & -5 \end{array} \right]$$

- 2a. Write the system of equations corresponding to each augmented matrix. $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ -5 & 7 & 2 \end{array} \right]$

$$\begin{cases} r_1: x-2y=1 \\ r_2: -5x+7y=2 \end{cases}$$

- 2b. Then perform the indicated row operation. $R_2 = -3r_1 + r_2$

$$\begin{array}{l} -3r_1 \rightarrow -3(x-2y) = -3(1) \\ \rightarrow -3x+6y = -3 \\ r_2 \rightarrow -5x+7y = 2 \\ \hline -8x+13y = -1 \rightarrow R_2 \end{array} \quad \begin{cases} x-2y=1 \\ -8x+13y=-1 \end{cases}$$

The reduced row echelon form of the system of linear equations is given. Write the system of equations corresponding to the given matrix. Determine whether the system is consistent or inconsistent. If it is consistent, give the solutions.

3. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 3 \end{array} \right]$ consistent

$$\begin{cases} x=5 \\ y=-8 \\ z=3 \end{cases}$$

Solution
 $(5, -8, 3)$

4. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 5 \end{array} \right]$ inconsistent

$$\begin{cases} x=2 \\ y=3 \\ 0=5 \leftarrow \text{False} \end{cases}$$

No solution

5. $\left[\begin{array}{ccc|c} 1 & 0 & 3 & 1 \\ 0 & 1 & -4 & 9 \\ 0 & 0 & 0 & 0 \end{array} \right]$ consistent

$$\begin{cases} x+3z=1 \rightarrow x=1-3z \\ y-4z=9 \rightarrow y=4z+9 \\ 0=0 \leftarrow \text{True} \end{cases}$$

$$\{(x, y, z) \mid x=1-3z, y=4z+9, z=\text{any real \#}\}$$

Solve each system of equations using matrices (row operations). If the system has no solution, say that it is inconsistent.

6. $\begin{cases} 2x+4y=10 \\ x+y=3 \end{cases}$

$$\left[\begin{array}{cc|c} 2 & 4 & 10 \\ 1 & 1 & 3 \end{array} \right] \xrightarrow[r_2 \rightarrow r_1]{r_1 \rightarrow R_2} \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & 4 & 10 \end{array} \right]$$

$$\xrightarrow{-2r_2 + r_1 \rightarrow R_2} \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 2 & 4 \end{array} \right] \xrightarrow{\frac{r_2}{2} \rightarrow R_2} \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & 2 \end{array} \right]$$

$$\begin{cases} x+y=3 \\ y=2 \end{cases} \quad \begin{cases} x+2=3 \\ x=1 \end{cases}$$

$(1, 2)$ solution

7. $\begin{cases} x-y=4 \\ 4x-4y=16 \end{cases}$

$$\left[\begin{array}{cc|c} 1 & -1 & 4 \\ 4 & -4 & 16 \end{array} \right]$$

$$\xrightarrow{\frac{r_2}{4} \rightarrow R_2} \left[\begin{array}{cc|c} 1 & -1 & 4 \\ 1 & -1 & 4 \end{array} \right]$$

$$\xrightarrow{r_1 - r_2 \rightarrow R_2} \left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & 0 & 0 \end{array} \right]$$

$$x-y=4$$

$$\{(x, y) \mid x=y+4, y=\text{any real \#}\}$$

$$10. \begin{cases} x+2y-3z=-16 \\ 2x-4y+z=20 \\ 3x+5y-2z=-17 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -16 \\ 2 & -4 & 1 & 20 \\ 3 & 5 & -2 & -17 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array} \right] \begin{array}{l} x=2 \\ y=-3 \\ z=4 \end{array}$$

type matrix in calculator
use rref (reduced row
echelon form)

Solution

(2, -3, 4)

$$12. \begin{cases} 3x-2y+2z=6 \\ 7x-3y+2z=-1 \\ 2x-3y+4z=0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & -2 & 2 & 6 \\ 7 & -3 & 2 & -1 \\ 2 & -3 & 4 & 0 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{2}{5} & 0 \\ 0 & 1 & -\frac{8}{5} & 6 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$$x - \frac{2}{5}z = 0$$

$$y - \frac{8}{5}z = 0$$

$0=1 \leftarrow$ Not
Possible
no solution

$$11. \begin{cases} x-y+z=5 \\ 3x+2y-2z=0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 5 \\ 3 & 2 & -2 & 0 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & -3 \end{array} \right] \begin{array}{l} x=2 \\ y-z=-3 \end{array}$$

consistent, dependent

Solution Set

$$\left\{ (x, y, z) \mid \begin{array}{l} x=2, y=z-3 \\ z \text{ any real \#} \end{array} \right\}$$

$$13. \begin{cases} x+2y-z=3 \\ 2x-y+2z=6 \\ x-3y+3z=4 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & -1 & 2 & 6 \\ 1 & -3 & 3 & 4 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & \frac{3}{5} & 0 \\ 0 & 1 & -\frac{4}{5} & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$$x + \frac{3}{5}z = 0$$

$$y - \frac{4}{5}z = 0$$

$0=1 \leftarrow$ Not
Possible

no solution