6.8 Practice Problems

1. The size P of a certain insect population at time (in days) obeys the function $P(t)=300e^{.05t}$.
a. Determine the number of insects at $t=0$ days.
b. What is the growth rate of the insect population?
c. Graph the function using the graphing utility.
d. What is the population after 10 days?
e. When will the insect population reach 1120?
f. When will the insect population double?

a) $P(0)=300e^{.05(0)}$
$=300e^{0}$
$=300(1)$
$=300$

b) $k=.05$

d) $P(10)=300e^{.05(10)}$
$=495.$

e) $\frac{1120}{300}=\frac{300e^{.05t}}{300}$
$\frac{1120}{300}=e^{.05t}$
$\frac{\ln\left(\frac{1120}{300}\right)}{.05}=\frac{.05t}{.05}$
$26.35=t$
days

f) $\frac{600}{300}=\frac{300e^{.05t}}{300}$
$2=e^{.05t}$
$\frac{\ln 2}{.05}=\frac{.05t}{.05}$
$13.86=t$
days

2. The population of Sarnun follows the exponential law. If the population doubled in size over 15 months and he current population is 10,000, what will the population be in 4 years from now?

$A=A_0e^{kt}$
$\frac{2A_0}{A_0}=\frac{A_0e^{k\cdot 15}}{A_0}$
$2=e^{15k}$
$\frac{\ln 2}{15}=\frac{15k}{15}$
$.0462=k$

$4\times 12=48$ months
$A=A_0e^{.046t}$
$A=10000e^{.046(48)}$
$=90975.$

3. The chemical element californium has a half-life of approximately 18 days. If 100 grams are present now, how much will be present in 60 days?

$A=A_0e^{kt}$
$\frac{\frac{1}{2}A_0}{A_0}=\frac{A_0e^{k(18)}}{A_0}$
$\frac{1}{2}=e^{18k}$
$\frac{\ln\frac{1}{2}}{18}=\frac{18k}{18}$
$-.039=k$

$A=100e^{-.039\cdot 60}$
$=9$ grams

4. After the release of a radioactive material into the atmosphere from a nuclear power plant in a country in 1992, the hay in that country was contaminated by a radioactive isotope (half-life 8 days). If it is save the feed the hay to the cows when 10% of the radioactive isotopes remains, how long did the farmers need to wait to use this hay?

$A = A_0 e^{kt}$

$\frac{\frac{1}{2}A_0}{A_0} = \frac{A_0 e^{k \cdot 8}}{A_0}$

$\frac{1}{2} = e^{8k}$

$\frac{\ln\frac{1}{2}}{8} = \frac{8k}{8}$

$k = -.087$

$\frac{.10 A_0}{A_0} = \frac{A_0 e^{-.087t}}{A_0}$

$.10 = e^{-.087t}$

$\frac{\ln .10}{-.087} = \frac{-.087t}{-.087}$

$26.5 = t$ days

5. The amount of caffeine in the human body has a half life of 4.9 hours to 6 hours depending on the person.

a. Assuming the caffeine half life for a particular person is 5 hours find the decay rate of caffeine and build an exponential model $\left[A = A_0 e^{kt}\right]$ for the decay rate. Round your answer to three decimal places.

b. Using the model in part a, find the amount of caffeine left in the body 1.5 hours after drinking a can of coke which contains 39.5 grams of caffeine. Round your answer to three decimal places.

a) $A = A_0 e^{kt}$

$\frac{1}{2}A_0 = A_0 e^{k \cdot 5}$

$\frac{1}{2} = e^{5k}$

$\frac{\ln(\frac{1}{2})}{5} = \frac{5k}{5}$

$-.139 = k$

b) $A = 39.5e^{-.139(1.5)}$

≈ 32.066

f) $2500 = \frac{5000}{1 + 125.22e^{-.364t}}$

$1 + 125.22e^{-.364t} = 2$

$\frac{125.22e^{-.364t}}{125.22} = \frac{1}{125.22}$

$e^{-.364t} = .007986$

$\frac{-.364t}{-.364} = \frac{\ln .007986}{-.364}$

$t = 13$ hours

6. The logistic growth model $P(t) = \frac{5000}{1 + 125.22e^{-0.364t}}$ represents the population (in grams) of a bacterium after t hours. Answer parts a. through f.

a. Determine the carrying capacity of the environment.
b. What is the growth rate of the bacteria?
c. Determine the initial population size.
d. What is the population after 8 hours?
e. When will the population be 4000 grams?
f. How long does it take or the population to reach one-half of the carrying capacity?

a) 5000

b) 36.4%

c) $P(0) = \frac{5000}{1 + 125.22e^{-.364(0)}}$

$= \frac{5000}{1 + 125.22} = \frac{5000}{126.22} \approx 40$

d) $P(8) = \frac{5000}{1 + 125.22e^{-.364(8)}}$

≈ 640

e) $4000 = \frac{5000}{1 + 125.22e^{-.364t}}$

$1 + 125.22e^{-.364t} = \frac{5000}{4000}$

$1 + 125.22e^{-.364t} = 1.25$
$-1 \qquad\qquad -1$

$\frac{125.22e^{-.364t}}{125.22} = \frac{.25}{125.22}$

$e^{-.364t} = .0019$

$\frac{-.364t}{-.364} = \frac{\ln(.0019)}{-.364}$

$t = 17$ hours