5.3 Practice Problems

1. Given information about a polynomial function $f(x)$ whose coefficients are real numbers. Find the remaining zeros of f. Then form a polynomial function having the given degree and zeros. Answers may vary depending on the choice of the leading coefficient.

   a. Degree: 3; zeros: $5, 3i$

remaining zero: $-3i$

$f(x) = a(x-5)(x+3i)(x-3i)$

$= a(x-5)(x^2+9)$

$= a(x^3-5x^2+9x-45)$

choose $a=1$

$f(x) = x^3-5x^2+9x-45$

f(x) is a polynomial w/ zeros of $5, -3i, 3i$

   b. Degree: 3; zeros: $-1, 2+i$

Remaining zero $2-i$

$f(x) = a(x+1)(x-(2+i))(x-(2-i))$

$= a(x+1)(x^2-4x+5)$

$= a(x^3-3x^2+x+5)$

Choose $a=1$

$f(x) = x^3-3x^2+x+5$

   c. Degree: 4; zeros 3 of multiplicity $2, 2+i$

Remaining zero $2-i$

$f(x) = a(x-3)^2(x-(2+i))(x-(2-i))$

$= a(x^2-6x+9)(x^2-4x+5)$

$= a(x^4-10x^3+38x^2-66x+45)$

choose $a=1$

$f(x) = x^4-10x^3+38x^2$
$-66x+45$

2. Use the zero to find the remaining zeros of the function.

   a. $f(x) = x^3+2x^2+25x+50$ zero: 5i $-5i$

$(x-5i)(x+5i) = x^2+25$

$$
\begin{array}{r|l}
 & x+2 \\
\hline
x^2+25 & x^3+2x^2+25x+50 \\
 & -x^3 \quad \mp 25x \\
\hline
 & 2x^2 \quad +50 \\
 & -2x^2 \quad \mp 50 \\
\hline
\end{array}
$$

$f(x) = (x-5i)(x+5i)(x+2)$

$= (x^2+25)(x+2)$

zeros: $5i, -5i, -2$

$(x^2-3x-10$

$(x-5)(x+2)$

b. $h(x)=x^4-9x^3+21x^2+21x-130$ zero: $3-2i$

other zero $3+2i$

$(x-(3-2i))(x-(3+2i))$

$= x^2-6x+13$

$$
\begin{array}{r}
x^2-3x+10 \\
x^2-6x+13 \overline{\big) x^4-9x^3+21x^2+21x-130} \\
-x^4+6x^3\mp13x^2 \downarrow \\
\hline
-3x^3+8x^2+21x \\
+3x^3\mp18x^2+39x \\
\hline
-10x+60x-130 \\
-10x+60x-130
\end{array}
$$

Zeros: $5, -2, 3+2i, 3-2i$

3. Find the complex zeros of each polynomial function. Write the function in factored form.

a. $f(x)=x^3-8$

$0=x^3-8$
$+8 \quad +8$
$8=x^3$
$\sqrt[3]{8}=\sqrt[3]{x^3}$
$2=x$

$$
\begin{array}{r|rrrr}
2 & 1 & 0 & 0 & -8 \\
 & \downarrow & 2 & 4 & 8 \\
\hline
 & 1 & 2 & 4 & 0
\end{array}
$$

x^2+2x+4

$f(x)=(x-2)(x^2+2x+4)$

$a=1 \quad b=2 \quad c=4$

$x=\frac{-2\pm\sqrt{2^2-4(1)(4)}}{2(1)}$

$=\frac{-2\pm\sqrt{4-16}}{2}$

$=\frac{-2\pm\sqrt{-12}}{2}=\frac{-2\pm\sqrt{-1\cdot4\cdot3}}{2}$

$=\frac{-2\pm2i\sqrt{3}}{2}$

$=-1\pm i\sqrt{3}$

$\boxed{\text{Zeros: } 2, -1+i\sqrt{3}, -1-i\sqrt{3}}$

b. $f(x)=x^4-16$

$f(x)=(x^2+4)(x^2-4)$

$=(x+2i)(x-2i)(x+2)(x-2)$

$\boxed{\text{Zeros: } -2i, 2i, -2, 2}$

c. $f(x)=x^4+3x^3-19x^2+27x-252$

$\frac{p}{q}=\pm1, \pm2, \pm3, \pm4, \pm6, \pm7, 9, \pm12, \pm14, \pm18, \pm21, \pm28, \pm36, \pm42, \pm63, \pm84, \pm126, \pm252$

from the graph zeros: $4, -7$

use synthetic or long div to reduce polynomial

$f(4)=4^4+3(4)^3-19(4)^2+27(4)-252$
$=256+192-304+108-252$
$=0$

4 is a zero

$$
\begin{array}{r|rrrrr}
4 & 1 & 3 & -19 & 27 & -252 \\
 & \downarrow & 4 & 28 & 36 & 252 \\
\hline
 & 1 & 7 & 9 & 63 & 0
\end{array}
$$

$x^3+7x^2+9x+63$

$f(-7)=(-7)^4+3(-7)^3-19(-7)^2+27(-7)-252$
$=2401-1029-931-189-252$
$=0$

-7 is a zero

$$
\begin{array}{r|rrrr}
-7 & 1 & 7 & 9 & 63 \\
 & \downarrow & -7 & 0 & -63 \\
\hline
 & 1 & 0 & 9 & 0
\end{array}
$$

$x^2+9=0$

$x^2=-9$

$x=\pm3i$

Zeros: $4, -7, 3i, -3i$