5.2 Practice Problems

1. Find the remainder if $f(x)=x^3+3x^2-x-3$ is divided by a) $x+2$ b) $x-1$

$f(-2)=(-2)^3+3(-2)^2-(-2)-3$
$= -8+3(4)+2-3$
$= -8+12+2-3$
$= 4+2-3$
$= 6-3$
$= 3$

The remainder when x^3+3x^2-x-3 is divided by $x+2$ is 3

$f(1)=(1)^3+3(1)^2-1-3$
$= 1+3-1-3$
$= 4-4$
$= 0$

The remainder when x^3+3x^2-x-3 is divided by $x-1$ is 0

2. Use the factor theorem to determine whether the function $f(x)=x^3+6x^2+8x-6$ has the factor
a) $x+2$ b) $x-1$

$f(-2)=(-2)^3+6(-2)^2+8(-2)-6$
$= -8+6(4)-16-6$
$= -8+24-16-6$
$= 16-16-6$
$= -6$

$x+2$ is not a factor of x^3+6x^2+8x-6

$f(1)=1^3+6(1)^2+8(1)-6$
$= 1+6+8-6$
$= 7+8-6$
$= 15-6$
$= 9$

$x-1$ is not a factor of x^3+6x^2+8x-6

3. Find the bounds to the zeros of each polynomial function.
a) $f(x)=x^3-5x^2-11x+11$ b) $f(x)=3x^3-2x^2+x+4$

factor out LC

a)
① $\max\{1, |-5|+|-11|+|11|\}$
$=\max\{1, 5+11+11\}$
$=\max\{1, 27\}=\boxed{27}$

② $1+\max\{|-5|,|-11|,|11|\}$
$=1+\max\{5,11,11\}$
$=1+11$
$=\boxed{12}$ ← choose lower #

$M=12$

$-12 \le \text{zeros} \le 12$

b)
$3\left(x^3-\frac{2}{3}x^2+\frac{1}{3}x+\frac{4}{3}\right)$

① $\max\{1, |-\frac{2}{3}|+|\frac{1}{3}|+|\frac{4}{3}|\}$
$=\max\{1, \frac{2}{3}+\frac{1}{3}+\frac{4}{3}\}$
$=\max\{1, \frac{7}{3}\}=\boxed{\frac{7}{3}}$

② $1+\max\{|-\frac{2}{3}|, |\frac{1}{3}|, |\frac{4}{3}|\}$
$=1+\max\{\frac{2}{3}, \frac{1}{3}, \frac{4}{3}\}$
$=1+\frac{4}{3}=\boxed{\frac{7}{3}}$

$M=\frac{7}{3}$

$-\frac{7}{3} \le \text{zeros} \le \frac{7}{3}$

4. Use the intermediate value theorem to show that polynomial function has a zero in the given interval.

$f(x)=x^4+8x^3-x^2+2;\quad [-1,0]$

$f(-1)=(-1)^4+8(-1)^3-(-1)^2+2$
$= 1-8-1+2$
$= -7-1+2$
$= -8+2 = -6$

$f(0)= 0^4+8(0)^3-0^2+2$
$= 2$

since f is a continuous function, $-1<0$ and $f(-1)$ is negative $f(0)$ is positive there is a zero between -1 and 0

5. Find the real zeros of the polynomial function $f(x)=x^3+3x^2-2x-6$

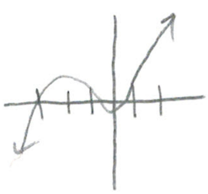
a) the function f has at most ___3___ zeros.

b) List the potential zeros using the rational zero theorem.

$p = \pm1, \pm2, \pm3, \pm6$
$q = \pm1$

$\frac{p}{q} = \pm1, \pm2, \pm3, \pm6$

c) Use a graphing utility to narrow your list of potential zeros.



The graph reveals an x-int that appears to cross at -3.

d) Use the factor theorem to determine if the potential rational zero is a zero.

$f(-3)=(-3)^3+3(-3)^2-2(-3)-6$
$= -27+27+6-6$
$= 0$

The function has a zero at -3 and a factor of $x+3$

e) If you find a zero, use synthetic division or long division to factor the polynomial.

$$\begin{array}{r} x^2 - 2 \\ x+3\,\overline{)\,x^3+3x^2-2x-6} \\ -x^3 \mp 3x^2 \downarrow \;\; \downarrow \\ \hline 0 - 2x - 6 \\ +2x+6 \\ \hline 0 \end{array}$$

$f(x) = (x+3)(x^2-2)$

f) Repeat until all of the zeros have been identified and the polynomial function is completely factored.

$f(x)=0$ ← to find remaining zeros

$(x+3)(x^2-2)=0$

$x+3=0 \quad x^2-2=0$
$-3\;\; -3 \quad x^2=2$
$x=-3 \quad \sqrt{x^2}=\sqrt{2}$
$x=\pm\sqrt{2}$

$f(x) = (x+3)(x-\sqrt{2})(x+\sqrt{2})$